PHOTON BUNCHING AND THE PHOTON-NOISE-LIMITED PERFORMANCE OF INFRARED DETECTORS

ROBERT W. BOYD

Institute of Optics, University of Rochester, Rochester, NY 14627, U.S.A.

Abstract—The photon-noise-limited performance of a radiation detector exposed to a thermal background of temperature T is treated by calculating the resulting specific detectivity, or D^*. Both ideal photon detectors of arbitrary quantum efficiency η and ideal thermal detectors of arbitrary emissivity η are considered. The effects of both shot noise and of excess, or photon-bunching, noise are included in the formalism. The relative contributions of these two sources depend on the quantum efficiency or emissivity of the detection system. For frequencies v such that $hv/kT \ll \eta$, excess noise can make an appreciable contribution to the total system noise. For the case of detection of narrow band radiation, D^* is independent of η in the limit $hv/kT \ll \eta$.

INTRODUCTION

The sensitivity of a radiation detector is limited most fundamentally by the randomness of photon arrival times at the detector, and a detector whose sensitivity is limited predominantly by this effect is said to be photon-noise-limited. In determining the photon-noise-limited performance of a detector exposed to radiation of thermal origin, it is necessary to take proper account of the Bose–Einstein nature of the radiation field. The photon occupancy (that is, the mean number of photons per field mode) of thermal radiation characterized by the excitation temperature T is known[1] to be of the form

$$\bar{n}_v = \frac{1}{e^{hv/kT} - 1} \tag{1}$$

where v is the frequency of field mode. Bose–Einstein statistics require that the mean-square fluctuation in the photon occupancy be given by[1]

$$(\Delta \bar{n}_v)^2 = \bar{n}_v(1 + \bar{n}_v). \tag{2}$$

In the limit $hv/kT \gg 1$, equation (1) shows that $\bar{n}_v \ll 1$, and therefore equation (2) becomes $(\Delta \bar{n}_v)^2 \simeq \bar{n}_v$. In the detection of such radiation, the individual photon-events are uncorrelated, and the noise in the detected signal can be considered to be a form of *shot noise*. However, in the limit $hv/kT \ll 1$, equation (1) shows that $\bar{n}_v \gg 1$, and equation (2) thus becomes $(\Delta \bar{n}_v)^2 \simeq \bar{n}_v^2$. The increased fluctuations in this limit are a consequence of photon bunching. The photons comprising the field are correlated in this case in such a way as to increase the noise in the detected signal above that given by shot noise. This increased noise has been called *excess noise*.

For the limiting case of detection by a system having unit quantum efficiency,[2] it is possible to establish a direct correspondence between the noise in the detected signal and the fluctuations in photon occupancy given by equation (2). In the more general case of a non-unit-quantum-efficiency detector, such a direct correspondence does not exist, because the randomness imposed by the detection process introduces additional noise, known as partition noise,[3] and also because the effect of photon bunching is decreased by the detection process.[4]

The purpose of the paper is to discuss the consequences of photon bunching on the photon-noise-limited performance of a detection system of arbitrary quantum efficiency. The performance is specified by calculating the specific detectivity, defined by

$$D^* = \frac{\sqrt{(A\Delta f)}}{P_N} \tag{3}$$

where A is the effective area of the detector, Δf is the electrical bandwidth of the detection system, and P_N is the noise equivalent power (NEP), defined as the incident signal power required to produce a signal-to-noise ratio of unity. The detector is assumed to be ideal in the sense that its output signal depends linearly on the incident power and in that only noise associated with the incident radiation contributes to the system noise. The analysis presented here should apply approximately to photovoltaic detectors or to cooled bolometers. The consequences of photon bunching to a photo-conductive detector are somewhat different, and have been discussed by van Vliet.[3] Several authors have previously addressed the problem discussed in this paper, but have obtained results differing somewhat from those presented here because they have either neglected to account for the dependence of the photon bunching contribution on quantum efficiency[5,6] or have concluded that photon-bunching effects are absent for the case of incoherent radiation.[7] The origin of these differences is discussed below in relation to equation (6) of this paper.

In the remainder of this paper, general expressions for the photon-noise-limited D^* of both photon and thermal detectors are derived, and these results are specialized to the two cases of a detector characterized by a constant quantum efficiency for all frequencies greater than some cut-on frequency and to the case of a detector that receives radiation only from a narrow spectral band. It is shown that in certain cases, which might well be encountered in the far infrared, photon bunching leads to a significant excess noise contribution to the detected signal. It is shown that the shot-noise and excess-noise contributions depend differently on the quantum efficiency of the detector. A surprising consequence of this effect is that, for $h\nu/kT \ll \eta$, the background-limited D^* of a narrow band detection system is independent of the system quantum efficiency.

PHOTON-NOISE-LIMITED PERFORMANCE OF PHOTON DETECTORS

Let us denote by Φ_ν the average rate at which photons contained in a narrow spectral interval $d\nu$ of the radiation background are incident on the detector. If the background is blackbody radiation of temperature T contained in a cone of half angle θ, this rate is given by[1]

$$\Phi_\nu = \frac{2\pi\nu^2 \sin^2\theta A \, d\nu}{c^2(e^{h\nu/kT} - 1)}.$$ (4)

The average rate at which photo-events occur is thus given by

$$N_\nu = \eta(\nu)\Phi_\nu,$$ (5)

where $\eta(\nu)$ denotes the quantum efficiency for radiation of frequency ν. It has been shown[3,4,8-10] that the mean-square fluctuation in this rate is given by

$$(\Delta N_\nu)^2 = 2N_\nu \Delta f(1 + \eta(\nu)\bar{n}_\nu).$$ (6)

The unity in the term in parentheses gives rise to the shot noise contribution whereas the second term gives rise to the excess-noise contribution. The relative contributions of these two terms depends on the quantum efficiency of the detection system, since a small quantum efficiency has the effect of lowering the degree of photon bunching.

Equation (6) is generally, but not universally, taken to be the correct expression for the fluctuation in the rate at which photoevents occur. It is identical, for instance, to equation (7.3) of Kingston[4] and to equation (68) of van Vliet.[3] The necessity of including the factor of η in equation (6) has been pointed out by Hanbury Brown and Twiss.[8] Hodara[9] and Ross[10] treat only the case of a detector with unity quantum efficiency, and equation (6) agrees with their results in this limit. Our equation (6) is somewhat in disagreement, however, with the analysis of reference 7 which replaces our factor $(1 + \eta\bar{n})$ with $(1 + \gamma\bar{n})$, where γ is a parameter describing the degree of coherence
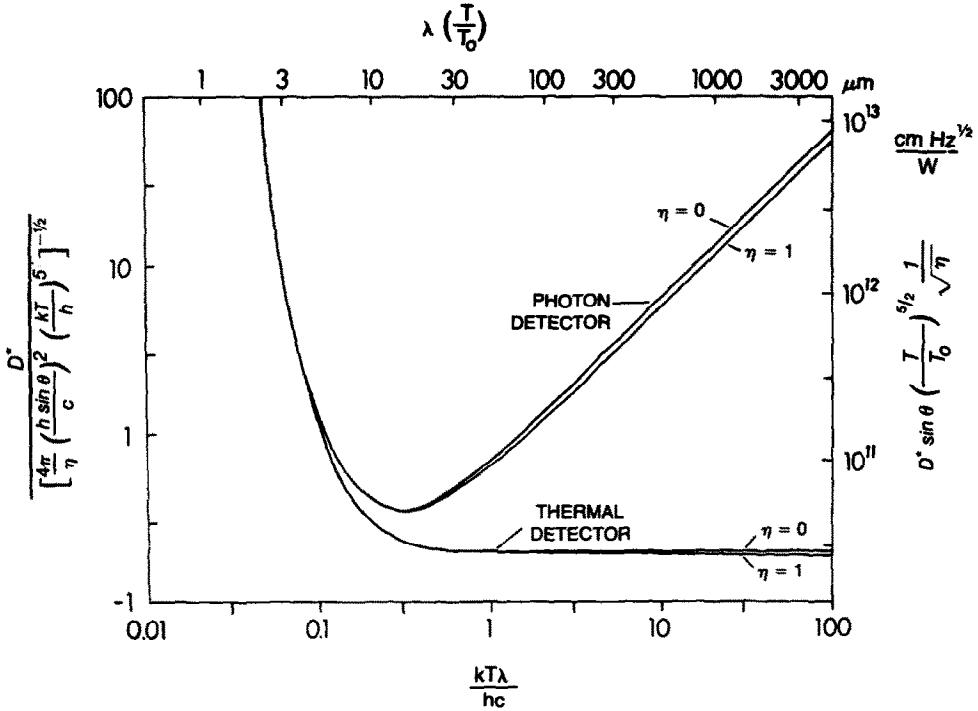
Fig. 1. Photon-noise-limited D^* of a photon detector and of a thermal detector. The detectors are exposed to thermal background radiation of temperature T, and are assumed to be characterized by constant quantum efficiencies (or emissivities) η for all wavelengths less than the cutoff wavelength λ of the detector. The upper abscissa and right-hand ordinate assume the value $T_0 \approx 300\text{K}$. The difference between the curves labeled $\eta = 1$ and $\eta = 0$ reflect the influence of photon bunching.

of the incident radiation. For the case of broad band radiation, the total fluctuation $(\Delta N)^2$ is given by integrating equation (6) over all frequencies.

If in addition to the background radiation, a signal of frequency v_s and power P_s falls onto the detector, additional photoevents occur at the rate $\eta(v_s) P_s/hv_s$. The NEP is determined by finding the value of P_s for which this rate is equal to the total fluctuation rate ΔN, and the value of D^* is then found using equation (3) as

$$D^* = \left[\frac{4\pi \sin^2\theta \, h^2 v_s^2}{\eta^2(v_s)c^2} \int_0^\infty \frac{\eta(v)v^2(e^{hv/kT} - 1 + \eta(v))\,dv}{(e^{hv/kT} - 1)^2} \right]^{-1/2}. \tag{7}$$

This general expression for the background-limited D^* of a photon detector can be simplified under special circumstances. Consider first the case in which $\eta(v)$ is equal to a constant value η for all frequencies greater than that of some cut-on frequency, which is assumed equal to the frequency v_s of the signal. Equation (7) then reduces to

$$D^* = \left[\frac{4\pi}{\eta} \left(\frac{h\sin\theta}{c} \right)^2 \left(\frac{kT}{h} \right)^5 \left(\frac{hv_s}{kT} \right)^2 \int_{hv_s/kT}^\infty \frac{x^2(e^x - 1 + \eta)\,dx}{(e^x - 1)^2} \right]^{-1/2}. \tag{8}$$

This expression has been evaluated numerically using Simpson's rule for $\eta = 1$ and $\eta \approx 0$, and the results are shown in Fig. 1. The lower abscissa is labelled in terms of the dimensionless parameter $kT/hv_s = kT\lambda/hc = 1/x$, while the upper abscissa has been labeled in terms of the wavelength in μm, assuming a blackbody temperature of $T_0 = 300\text{K}$. This scale can be used at other temperatures T by dividing the values by T/T_0. The left-hand ordinate gives D^* in the indicated dimensionless units, while the right-hand ordinate gives D^* in conventional units for the case $T_0 = 300\text{K}$, $\eta = 1$, and $\sin\theta = 1$. To use this scale under other circumstances, the values should be divided by $\sin\theta(T/T_0)^{5/2}\,\eta^{-1/2}$. The dominant effect of varying η is to lower D^* by a factor

of $\eta^{-1/2}$. For this reason, $D^*/\sqrt{\eta}$ has been plotted in Fig. 1. In addition, the overall shape of the curve depends on η. For $hv \ll kT$, the curve for $\eta = 0$ is 17% higher than that for $\eta = 1$. This difference can be understood as a consequence of photon bunching, which is absent in the limit $\eta \to 0$ since there is then no possibility that the photo-events can be correlated.

Another special case, and one for which the effects of photon bunching are much more pronounced, is that in which the detector is shielded from background radiation by a narrow bandpass filter, whose central frequency v is related to its bandwidth Δv by $\Delta v = v/Q$. For $Q \gg 1$, equation (7) reduces to

$$D^* = \left[\frac{4\pi}{\eta Q} \left(\frac{h \sin \theta}{c} \right)^2 v^5 \frac{e^{hv/kT} - 1 + \eta}{(e^{hv/kT} - 1)^2} \right]^{-1/2} \tag{9}$$

where $\eta = \eta(v)$. This equation can be written in terms of the dimensionless parameter $x = hv/kT$ as

$$D^* = \left\{ \frac{4\pi}{Q} \left(\frac{h \sin \theta}{c} \right)^2 \left(\frac{kT}{h} \right)^5 \left[\frac{x^5}{\eta} \frac{(e^x - 1 + \eta)}{(e^x - 1)^2} \right] \right\}^{-1/2}. \tag{10}$$

This functional form has been plotted vs $1/x$ in Fig. 2 for the case $\eta = 1, 0.1, 0.01,$ and 0.001. Here the left-hand ordinate gives D^* in terms of the indicated dimensionless units, while the right-hand ordinate gives D^* in conventional units for the case $Q_0 = 100$ and $T_0 = 300K$. To use the right hand scale under general circumstances, the values should
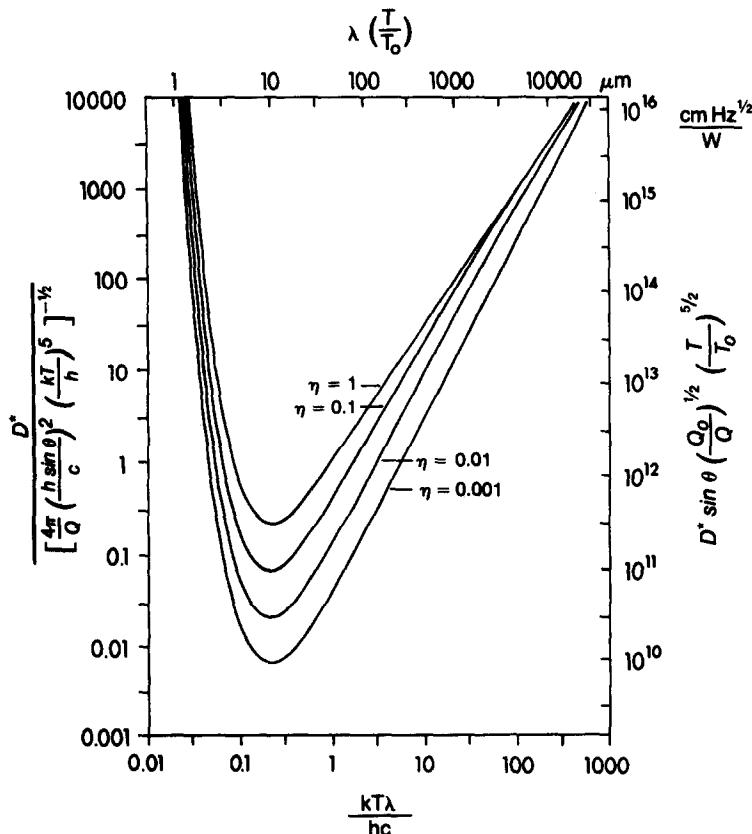

Fig. 2. Photon-noise-limited D^* of either a photon or a thermal detector. The detectors are exposed to thermal background radiation of temperature T, and respond with quantum efficiency (or emissivity) η only within a narrow spectral interval of width $\Delta v = v/Q$, where $v = c/\lambda$ is the central frequency of the band. The upper abscissa and right-hand ordinate assume the values $Q_0 = 100$ and $T_0 = 300K$. For $hv \ll kT$, the shapes of these curves depend on η, illustrating the influence of phonon bunching.

be divided by $\sin \theta (Q_0/Q)^{1/2}(T/T_0)^{5/2}$. For $h\nu/kT \ll \eta$, D^* is proportional to $\sqrt{\eta}$, as would be expected from shot noise; however for $h\nu/kT \ll \eta$, D^* is independent of the quantum efficiency and attains the value

$$D^* = \left[\frac{4\pi}{Q} \left(\frac{h \sin \theta}{c} \right)^2 \left(\frac{kT}{h} \right)^5 \left(\frac{h\nu}{kT} \right)^3 \right]^{-1/2}. \tag{11}$$

In this limit, excess noise is much larger than shot noise, and thus D^* is independent of the frequency of photoevents.

PHOTON-NOISE-LIMITED PERFORMANCE OF THERMAL DETECTORS

The arguments presented in the preceding section have to be modified in order to treat thermal detectors, since these devices respond to the power, and not the photon flux, incident upon them. It is assumed that the detector is cooled so that thermal fluctuations in the power emitted by the detector are negligible. The fluctuation in photo-event rate given by equation (6) implies a fluctuation in absorbed power in the frequency interval $d\nu$ given by

$$\begin{aligned}
(\Delta P_\nu)^2 &= h^2\nu^2(\Delta N_\nu)^2 \\
&= 2h^2\nu^2\eta(\nu)\Phi_\nu\Delta f(1 + \eta(\nu)\bar{n}_\nu). \tag{12}
\end{aligned}$$

The total power fluctuation is given by integrating this expression over all frequencies. The value of D^* is obtained by letting $P_N = \Delta P$ in equation (3) and using equation (1) and (4) to obtain

$$D^* = \left[\frac{4\pi \sin^2\theta h^2}{\eta^2(\nu_s)c^2} \int_0^\infty \frac{\eta(\nu)\nu^4(e^{h\nu/kT} - 1 + \eta(\nu)) d\nu}{(e^{h\nu/kT} - 1)^2} \right]^{-1/2}. \tag{13}$$

The quantity $\eta(\nu)$ can still be considered as a quantum efficiency, although it is more conventional to interpret $\eta(\nu)$ as the absorbtivity, or equivalently as the emissivity, of the detector element.

Equation (13) can be simplified for the same cases as that of the photon detector. For the case $\eta(\nu) = \eta$ for $\nu > \nu_s$, and $\eta(\nu) = 0$ otherwise, D^* is given by

$$D^* = \left[\frac{4\pi}{\eta} \left(\frac{h \sin \theta}{c} \right)^2 \left(\frac{kT}{h} \right)^5 \int_{h\nu_s/kT}^\infty \frac{x^4(e^x - 1 + \eta)}{(e^x - 1)^2} dx \right]^{-1/2}. \tag{14}$$

This expression has also been evaluated numerically, and the results are also shown in Fig. 1 for the cases $\eta = 1$ and $\eta \approx 0$. For $h\nu/kT \ll 1$, the curves approach constant values, and the curve with $\eta \approx 0$ is only 2% higher than that with $\eta = 1$ for this case. In the other special case of detection of a narrow spectral band, the results are identical to those for the case of a photon detector, discussed earlier and shown in Fig. 2.

REFERENCES

1. LANDAU L. D. & E. M. LIFSHITZ, *Statistical Physics*, pp. 351–357. Pergamon Press, Oxford (1958).
2. The quantum efficiency will be defined for both thermal and photon detectors to be the ratio of the number of detected photons to the number of photons incident on the detection system. Transmission losses in the optical components of the detection system thus decrease the quantum efficiency.
3. VAN VLIET K. M., *Appl. Opt.* **6**, 1145 (1967).
4. KINGSTON R. H., *Detection of Optical and Infrared Radiation*, Section 7.1. Springer, New York (1978).
5. KRUSE P. W., L. D. MCGLAUCHLIN & R. B. MCQUISTAN, *Elements of Infrared Technology*, Section 9.4. Wiley, New York (1962).
6. JACOBS S. F. & M. SARGENT, III, *Infrared Phys.* **10**, 223 (1970).

7. Wolfe W. L. & G. J. Zissis, *The Infrared Handbook*, pp. 1–14 and 11–39 through 11–44. ONR, Washington (1978).
8. Hanbury Brown R. & R. Q. Twiss, *Proc. R. Soc. A* **242**, 300 (1957); see, especially, equations (2.22) and (3.43) through (3.46).
9. Hodara H., *Proc. IEEE* **53**, 696 (1965); see, especially, equation (5b).
10. Ross M., *Laser Receivers*, pp. 14–23, Wiley, New York (1966); see, especially, equation (2.20).