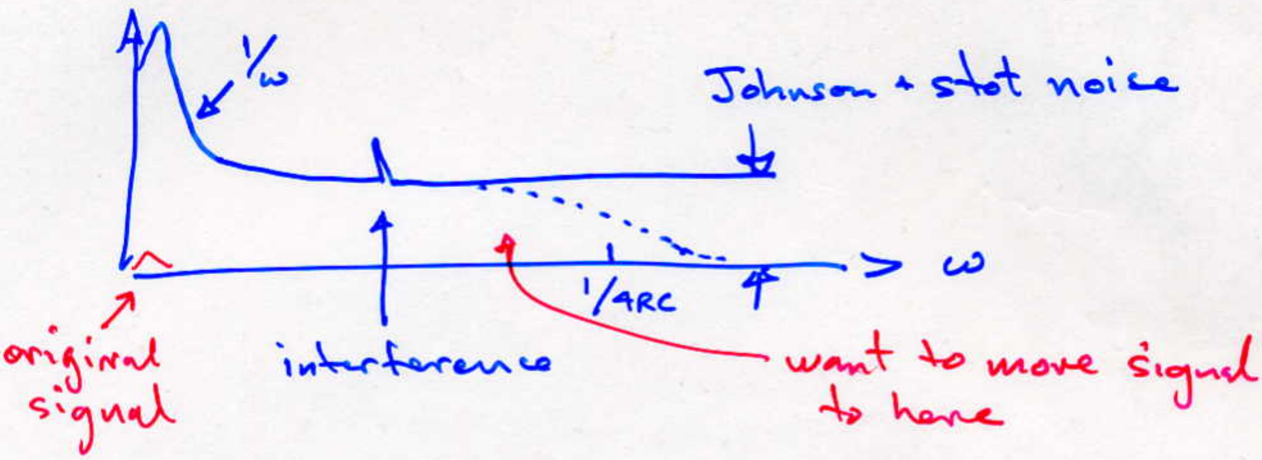
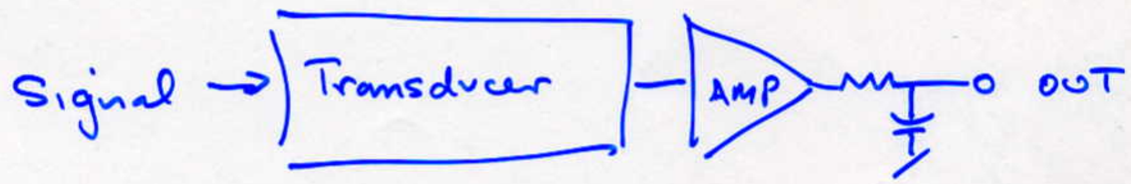
Review of noise sources

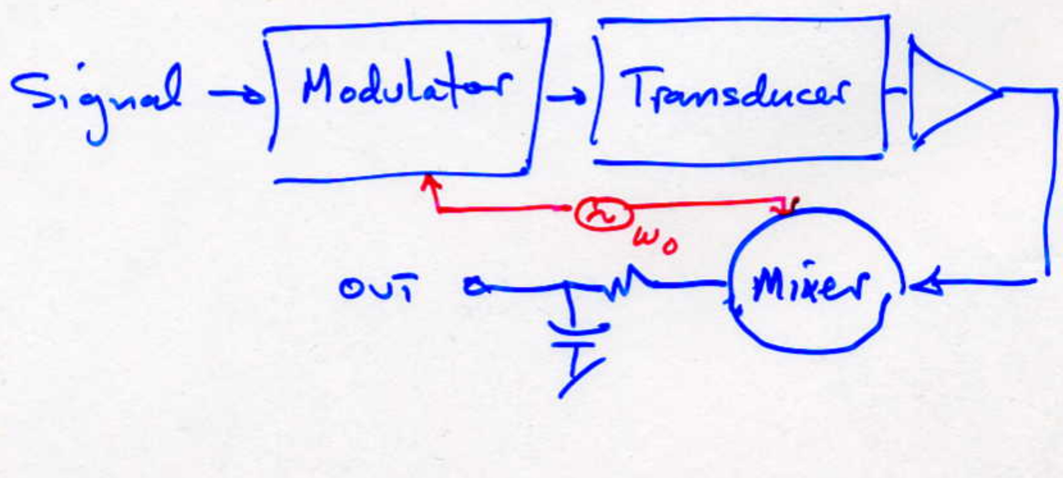


If signal is near ω = 0, we must devise a method of avoiding 1/ω noise and interference at 60 Hz, etc.

BAD:

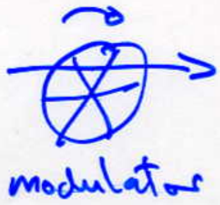


BETTER: move signal to high frequency

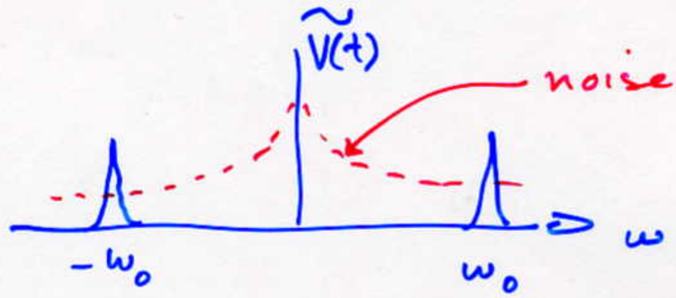


Assume :

signal → modulator → signal $\cdot e^{i\omega_0 t}$ $\quad V(t)$



← Reference ω_0 $\quad R(t)$

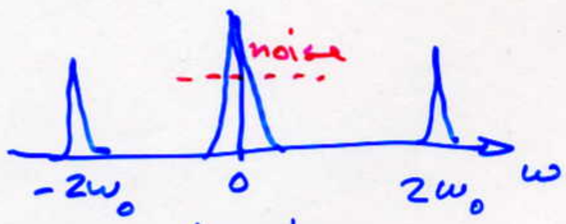


Mixer multiplier : $\quad V(t) \cdot R(t)$

If $V(t)$ and $R(t)$ are in phase :

and the Fourier transform

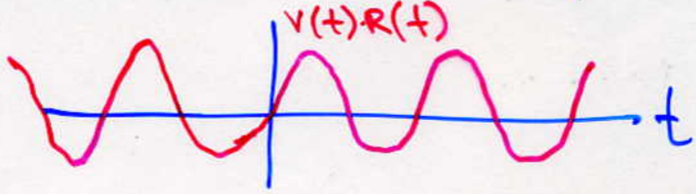
$\widetilde{V(t)} * \widetilde{R(t)}$



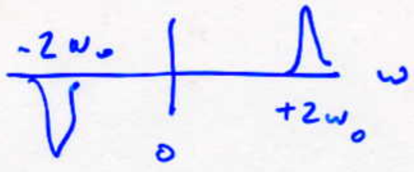
Low pass filter $\quad \Delta f \sim 1/4RC$

Harmonics at $\pm 2\omega_0$ are rejected.
Output is constant voltage ($\omega = 0$ peak).

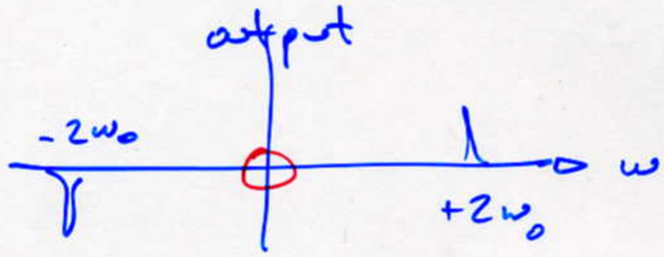
If $V(t)$ and $R(t)$ are 90° out of phase



$\tilde{R}(t)$



then $\tilde{V}(t) * \tilde{R}(t)$ has no $\omega = 0$ output



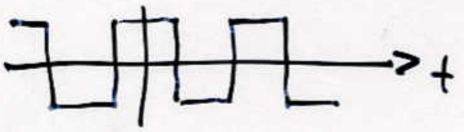
Noise near $\pm\omega_0$ is shifted to $\omega = 0$ and $\pm 2\omega_0$ by the mixer. The low pass filter selects an effective bandwidth $\Delta f = 1/4RC \ll 2\omega_0$.

Signal averaging can take place at a low noise part of the spectrum

Good experiments do this more than once: nested chops.

Real lock-in amplifier conditions the reference signal to avoid amplitude noise on the reference.

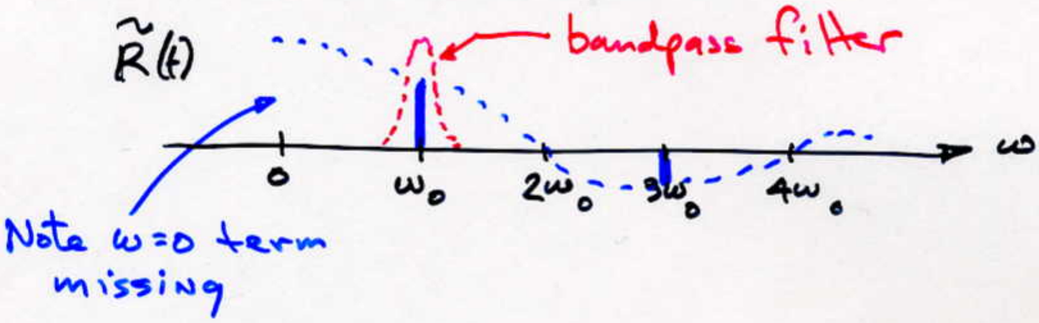
One method: amplify + clip reference signal



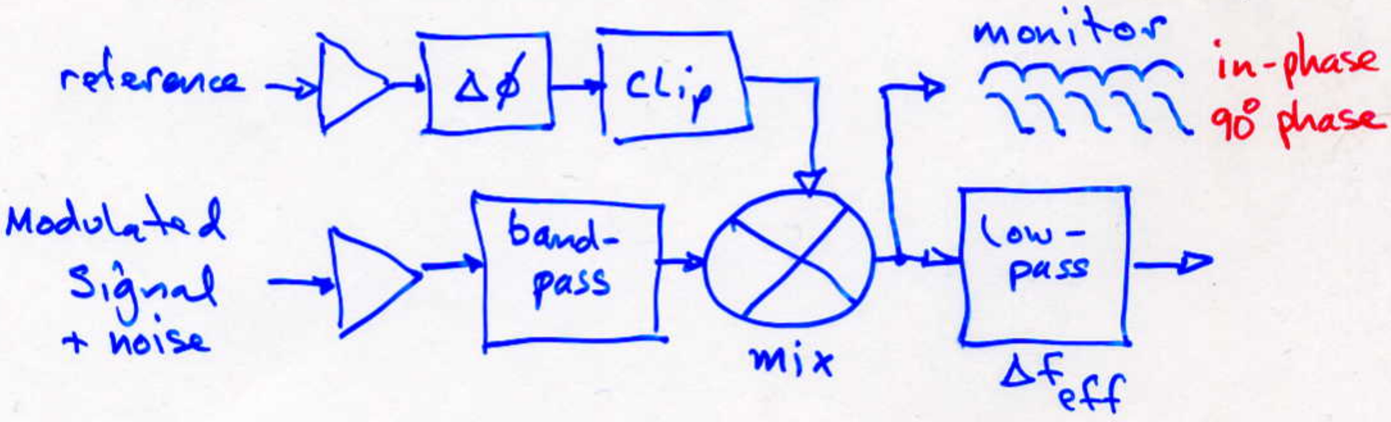
zero average

has Fourier transform → odd harmonics

To avoid adding in noise from all harmonics, band-pass filter before mixer.



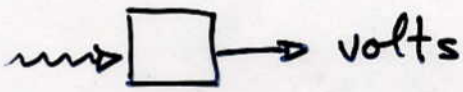
Summary: A lock-in amplifier (PSD) is a linear signal demodulator which requires a reference input.



Can be done digitally if reference and signal are adequately sampled.

Systematic error

Systematics can and do occur at multiple stages of an experiment:

- Interface with nature. The sample or probe is not what you thought it was.
- The link between nature and the detector / transducer.
- The transducer 

- The experimental apparatus
- The execution of the experiment.
- The interpretation of the data.
- Theoretical models as priors in data analysis.

Most of these errors may be avoided, and the rest may be detected.

CHOP THE SIGNAL

We saw that chopping is a way of avoiding noise, by moving the signal spectrum and by averaging. But chopping also overcomes many forms of systematic error.

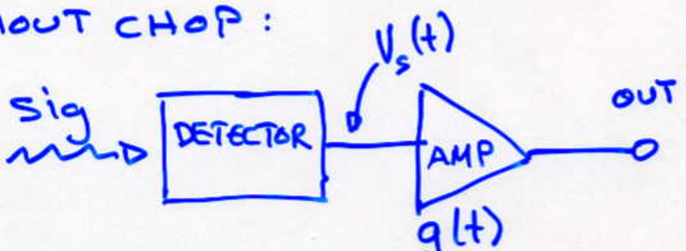
Most systems (electrical, mechanical) drift slowly. i.e. their transfer function or gain varies.

Such drifts occur in the link to the detector, and within the experiment. They generally have a $1/f$ spectrum!

$\therefore$ Chop the signal as close to nature as possible

It is helpful to examine two cases, with and without chopping. We shall assume zero noise.

WITHOUT CHOP:

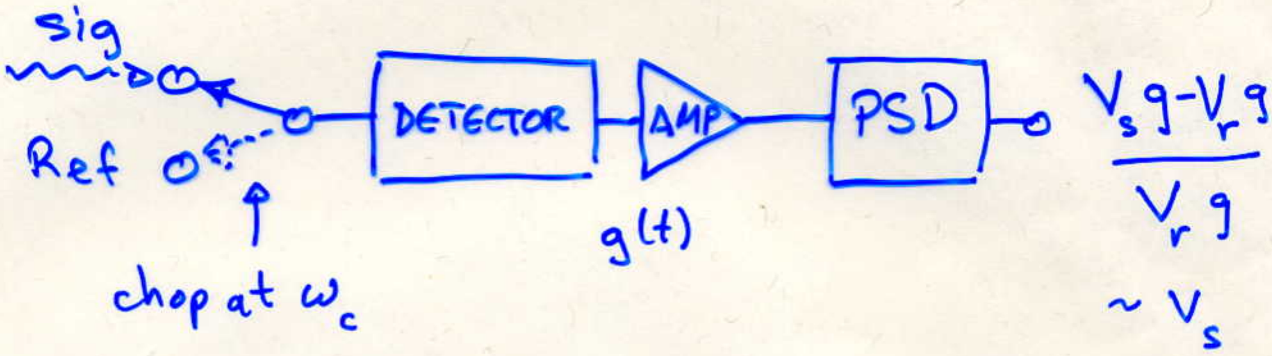


$$V(t) = V_s(t) \cdot g(t)$$

$$V_{AV} = \int_0^{\tau} V_s \, g \, dt$$

If gain drift is 1% $0 < t < \tau$
then systematic error in $V_{AV} \sim 1\%$

WITH CHOP TO REFERENCE:



If the chop took place after the detector, then drift in the detector would appear as a drift in the signal.

Since the sources of variation of g(t) in a system are hard to find and compensate, it is easier to chop the input to a reference.

There will be several examples of this later in the course, most notably the detection of the cosmic microwave background radiation.

Precision experiments often include multiple chops in the time domain.

In the above example, one could chop or modulate the signal itself at some lower rate $\omega_0 << \omega_c$. i.e. vary some parameter.