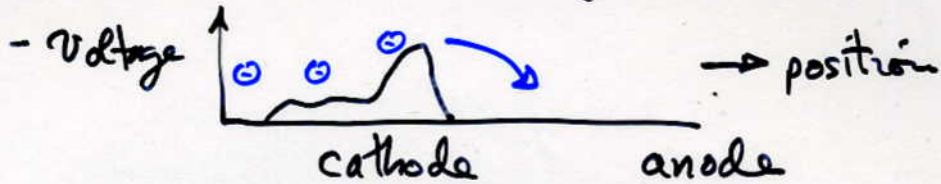


Shot noise

Charge is quantized.

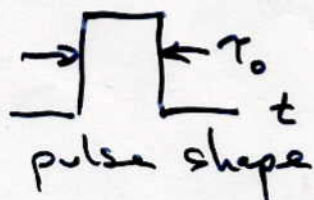
Consider current through barrier =



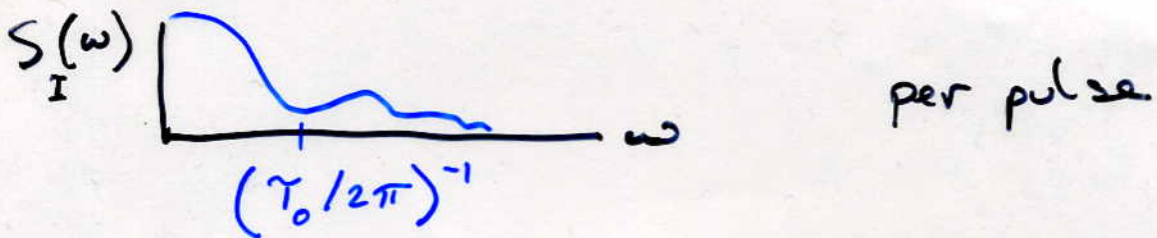
Auto correlation function for δ function spikes is $I(t) I(t+\tau) = \delta(\tau)$

According to the WK theorem we should get white noise.

However, if the pulses have finite width τ_0 , like charge transiting a diode, then the auto correlation function looks like



WK theorem $\rightarrow S_I(\omega) \sim \left[\frac{\sin \omega \tau_0 / 2}{\omega \tau_0 / 2} \right]^2$



Poisson statistics for random events:
 Given n events in time τ (note $\bar{n} \neq 0$)

$$\overline{(\Delta n)^2} = \bar{n}$$

Since electrons are charged, there is a current $I = \bar{n} e / \tau$

$$\therefore \overline{(\Delta I)^2} = \frac{\overline{(\Delta n)^2} e^2}{\tau^2} = \bar{n} e^2 / \tau^2$$

$$\text{or } \overline{\Delta I^2} = e I / \tau \equiv S_I(f) \Delta f_{\text{eff}}$$

Averaging over time $\tau \rightarrow$ effective bandwidth $\frac{1}{2\tau}$

$$\Rightarrow \boxed{S_I(f) = 2e\bar{I}} \quad \text{SHOT NOISE}$$

Constant white noise up to frequency cutoff related to the pulse width τ_0 .

Unlike Johnson noise, this is not thermal.

$$1 \text{ amp} \rightarrow 5.7 \text{ nA} / \text{kHz}$$

1/f Noise

$$S(\omega) \sim \omega^{-1}$$

Very common. No universal detailed single mechanism.

2 Examples:

- Distribution of length of power failures
- Spectral intensity (loudness) of speech or music of people, whales ...

Useful diagnostic test: Look at noise on oscilloscope. Change the time base. Noise looks the same. Why?

Assume the "eye" measures $\Delta V_{\text{rms}} = S_V(f) \Delta f$ over time τ , where $\tau \equiv$ sweep time.

Bandwidth $\sim 1/\tau$ The characteristic frequencies displayed are $\sim 1/\tau$.

If $S_V(f) \sim f^{-1} \sim \tau$, then $S_V(f) \Delta f$ is independent of τ .

Generally, sources of "1/f noise" have power spectrum $S_V(f) \sim f^{-\alpha}$

$$\frac{1}{2} < \alpha < 2$$

Example: Flicker noise

Random modulation of barrier to charge transfer.

- Fluctuations in conductance between carbon granules in carbon resistors.
- Fluctuations in emission rate of electrons from a filament (cathode patch effect).
- Fluctuations in tunneling probability in superconducting junction due to trapping of electrons in the barrier.

"Telegraph noise"



If the process has a relaxation time τ , the correlation function of noise has the form $e^{-t/\tau}$, and power spectrum

$$S_I(f) \sim (1 + \omega^2 \tau^2)^{-1} \sim \omega^{-2}$$

One way to obtain a ω^{-1} spectrum is to average over a range of relaxation times.

* $1/f$ noise cannot be eliminated by averaging longer!

Origins of $1/f$ noise

- resistance fluctuations semiconductors
- mobility or carrier density fluctuations

Generally, $1/f$ occurs when a process requires the successful completion of many sub-processes or tasks

(Montrol + Shlesinger PNAS 79 3380, 1982)

then probability P : $\log P = \log P_1 + \log P_2 + \dots + \log P_N$
results in a Log-normal distribution.

$\log P$ is normally distributed.

→ Scale invariant ($1/f$) distribution if dispersion σ is large. [over some range in f]

Consider processes with correlation times τ_i .

Each has spectrum $S(\omega) \sim e^{-t/\tau_i} \sim \tau_i / (1 + \omega^2 \tau_i^2)$

A scale-invariant weighting function $P(\tau) d\tau \sim d\tau / \tau$

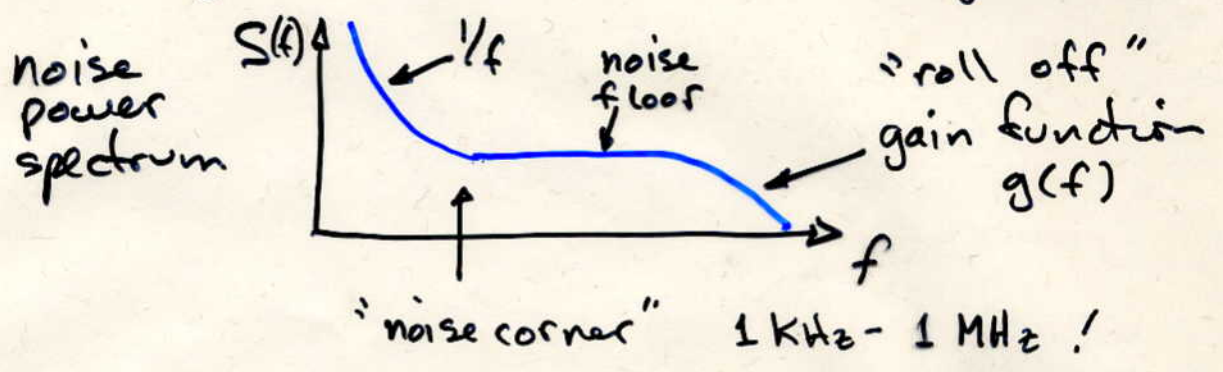
Then gives a total power spectrum:

$$\int_{\tau_1}^{\tau_2} S_{\tau}(\omega) P(\tau) d\tau \sim \int_{\tau_1}^{\tau_2} \frac{\tau}{(1 + \omega^2 \tau^2)} \frac{d\tau}{\tau} \sim \frac{\tan^{-1} \omega \tau}{\omega} \Big|_{\tau_1}^{\tau_2}$$

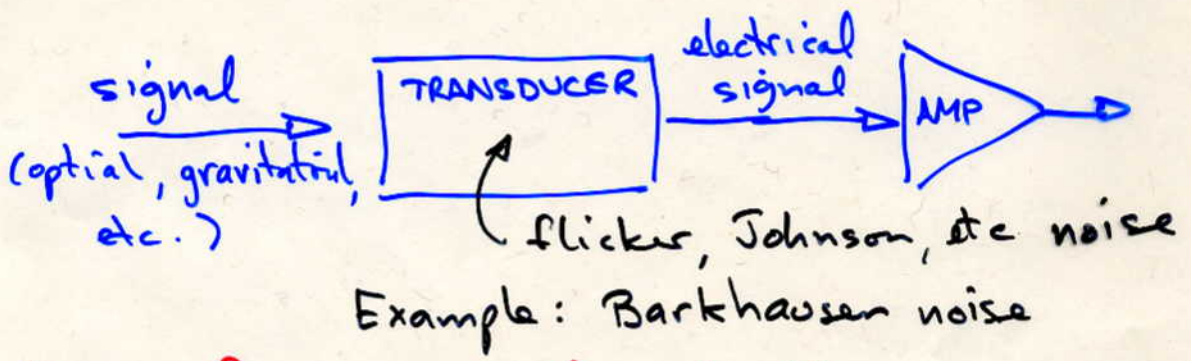
If $\tau_2 / \tau_1 \gg 1$, spectrum $\sim 1/\omega$ over large range.

- Note pure $1/\omega$ spectrum has log divergent integral

Real amplifiers have $1/f$ noise.
 The very best have noise floor at $f > 1000 \text{ Hz}$ in the nanovolt/ $\text{Hz}^{1/2}$ range, with larger noise at lower frequencies:

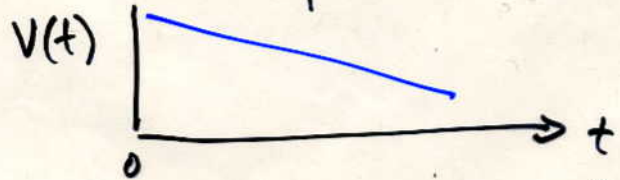


Even Transducers have $1/f$ noise !



Not all low-frequency noise is $1/f$:

Consider a steady drift (a type of systematic error)



Can be represented as convolution $\square * \square$

Fourier transform = square of $\int \square \sim \left(\frac{2}{\omega} \sin \frac{\omega \tau}{2} \right)^2 \sim \frac{1}{\omega^2}$

