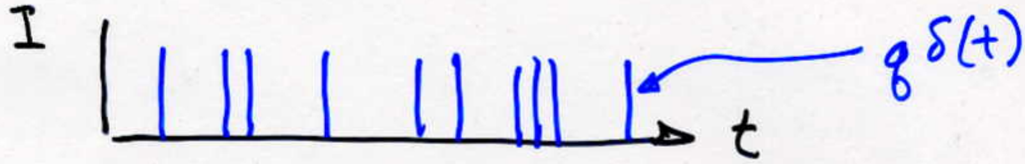
Shot noise

Charge is quantized.

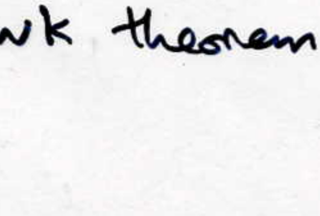
Consider current through barrier:

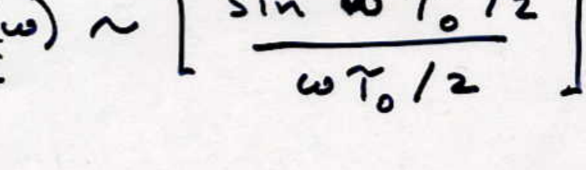


Autocorrelation function for δ function spikes is $\overline{I(t)\,I(t+\tau)} = \delta(t)$

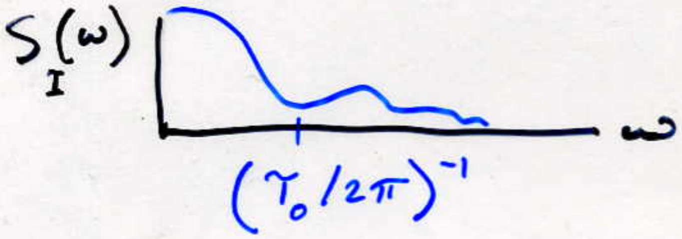
According to the WK theorem we should get white noise.

However, if the pulses have finite width τ_0, like charge transiting a diode, then the autocorrelation function looks like





WK theorem → $S_I(\omega) \sim \left[\dfrac{\sin \omega\tau_0/2}{\omega\tau_0/2}\right]^2$



per pulse

Poisson statistics for random events:
Given n events in time τ (note $\bar{n} \neq 0$)

$$\overline{(\Delta n)^2} = \bar{n}$$

Since electrons are charged, there is a current $I = \bar{n} e / \tau$

$$\therefore \overline{(\Delta I)^2} = \frac{\overline{(\Delta n)^2} e^2}{\tau^2} = \bar{n} e^2 / \tau^2$$

$$\text{or} \quad \overline{\Delta I^2} = e \bar{I} / \tau \quad \equiv S_I(f) \, \Delta f_{eff}$$

Averaging over time τ $\rightarrow$ effective bandwidth $\frac{1}{2\tau}$

$$\Rightarrow \boxed{S_I(f) = 2 e \bar{I}} \qquad \text{SHOT NOISE}$$

Constant white noise up to frequency cutoff related to the pulse width τ_0.

Unlike Johnson noise, this is not thermal.

1 amp $\rightarrow$ 5.7 nA / kHz

1/f Noise

$S(\omega) \sim \omega^{-1}$

Very common. No universal detailed single mechanism.

2 Examples:

- Distribution of length of power failures
- Spectral intensity (loudness) of speech or music of people, whales ...

Useful diagnostic test: Look at noise on oscilloscope. Change the time base. Noise looks the same. Why?

Assume the "eye" measures $\Delta V_{rms} = S_v(f)\,\Delta f$ over time τ, where $\tau \equiv$ sweep time.

Bandwidth $\sim 1/\tau$ The characteristic frequencies displayed are $\sim 1/\tau$.

If $S_v(f) \sim f^{-1} \sim \tau$, then $S_v(f)\,\Delta f$ is independent of τ.

Generally, sources of "1/f noise" have power spectrum $S_v(f) \sim f^{-\alpha}$

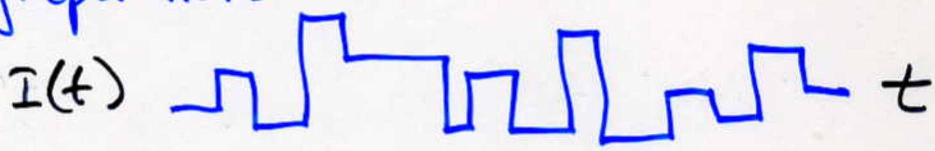
$$\frac{1}{2} < \alpha < 2$$

Example: Flicker noise

Random modulation of barrier to charge transfer.

- Fluctuations in conductance between carbon granules in carbon resistors.
- Fluctuations in emission rate of electrons from a filament (cathode patch effect).
- Fluctuations in tunneling probability in superconducting junction due to trapping of electrons in the barrier.

"Telegraph noise"



If the process has a relaxation time τ, the correlation function of noise has the form $e^{-t/\tau}$, and power spectrum

$$S_I(f) \sim (1 + \omega^2\tau^2)^{-1} \sim \omega^{-2}$$

One way to obtain a ω^{-1} spectrum is to average over a range of relaxation times.

* 1/f noise cannot be eliminated by averaging longer!

Origins of 1/f noise

- resistance fluctuations — semiconductors
- mobility or carrier density fluctuations

Generally, 1/f occurs when a process requires the successful completion of many sub-processes or tasks (Montroll + Shlesinger PNAS 79 3380, 1982)

then probability P: $\log P = \log P_1 + \log P_2 + \dots \log P_N$

results in a Log-normal distribution.

$\log P$ is normally distributed.

→ Scale invariant (1/f) distribution if dispersion σ is large. [over some range in f]

Consider processes with correlation times τ_i.

Each has spectrum $S(\omega) \sim \widetilde{e^{-t/\tau_i}} \sim \tau_i/(1+\omega^2\tau_i^2)$

A scale-invariant weighting function $P(\tau)d\tau \sim d\tau/\tau$

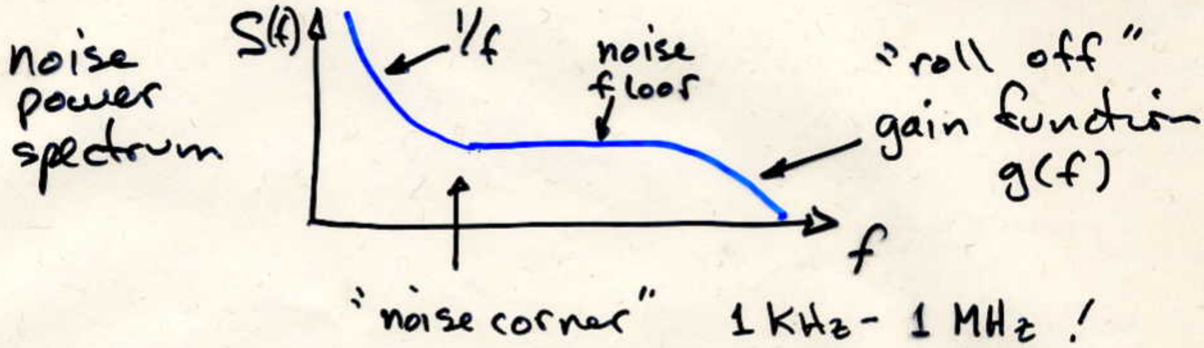
Then gives a total power spectrum:

$$\int_{\tau_1}^{\tau_2} S_\tau(\omega)\, P(\tau)\, d\tau \sim \int_{\tau_1}^{\tau_2} \frac{\tau}{(1+\omega^2\tau^2)} \frac{d\tau}{\tau} \sim \left.\frac{\tan^{-1}\omega\tau}{\omega}\right|_{\tau_1}^{\tau_2}$$

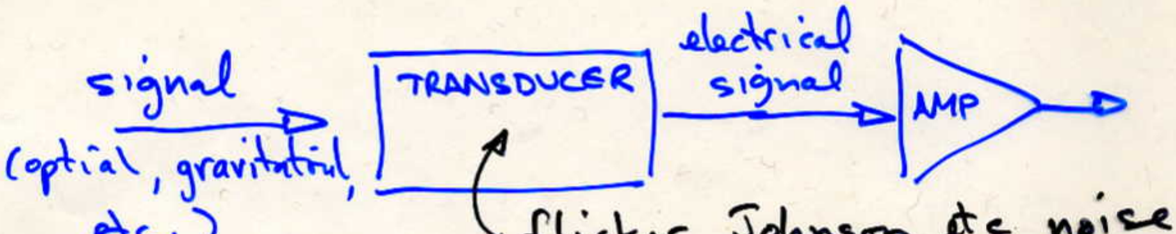
If $\tau_2/\tau_1 \gg 1$, spectrum $\sim 1/\omega$ over large range.

- Note pure $\frac{1}{\omega}$ spectrum has log divergent integral

Real amplifiers have $1/f$ noise.
The very best have noise floor at $f > 1000$ Hz in the nanovolt/$\text{Hz}^{1/2}$ range, with larger noise at lower frequencies:



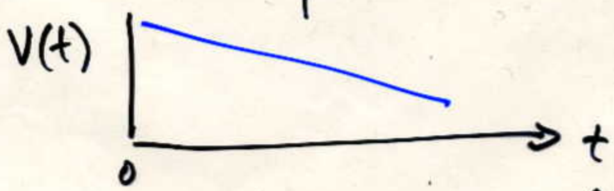
Even Transducers have $1/f$ noise!



Example: Barkhausen noise

Not all low-frequency noise is $1/f$:

Consider a steady drift (a type of systematic error)



Can be represented as convolution $\sqcap * \sqcap \quad t$

Fourier transform = square of $\widetilde{\sqcap} = \left(\frac{2}{\omega}\sin\frac{\omega\tau}{2}\right)^2 \sim \frac{1}{\omega^2}$

In semiconductors, $1/f$ noise depends on size and bias voltage:

$$S_V(\omega) \sim \omega^{-\alpha} V_{bias}^{2} \, size^{-1}$$ "dirt effect"

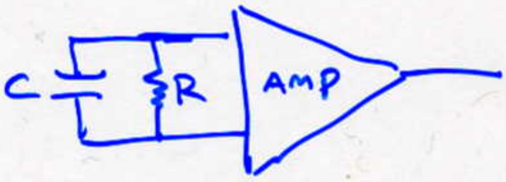
kTC noise

C 

$\overline{V_{out}^2} \sim \frac{kT}{C}$

Appears to violate Fluctuation-Dissipation Theorem.

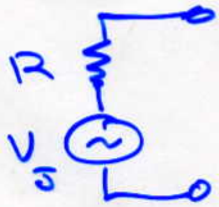
Real devices have resistance:



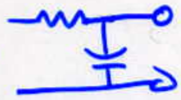
Energy in capacitor is $\frac{1}{2}CV^2 = \frac{1}{2}kT$

by equipartition $\overline{V^2} = kT/C$

Equivalent calculation:



$V_J^2 = 4kTR\,\Delta f$

Effective bandwidth  $\frac{1}{4RC}$

$\therefore \overline{V_{out}^2} = 4kTR/4RC = kT/C$

- R cancels out because R both creates noise and defines bandwidth.

Signal/Noise Ratio (want to maximize!)

In general, the signal and noise have different spectra (or can be made to differ).

Thus, we can attempt to move the signal to a part of the spectrum where the noise is low.

Noise, by definition, has random phase.

By averaging over time, we can increase the S/N ratio:

$$S \sim \int^{\tau} \text{signal}\, dt \sim \tau$$

$$N \sim \int^{\tau} \text{noise}\, dt \sim \tau^{1/2}$$

$$\frac{S}{N} \sim \tau^{1/2}$$ "signal averaging"

Example: Poisson noise + signal

Variance of noise $\overline{\delta V_n^2} \sim \overline{V_n}$

Integrate for time $\tau \rightarrow \overline{V_n}\tau = \overline{\delta V_n^2}$

so $\delta V_{rms}^{noise} \sim \tau^{1/2}$

whereas signal $\sim \tau$ $\quad S/N \sim \tau^{1/2}$

strictly true only if we can make signal repeat i.e. signal averaging works only if we can make the signal repeat many times. N samples.

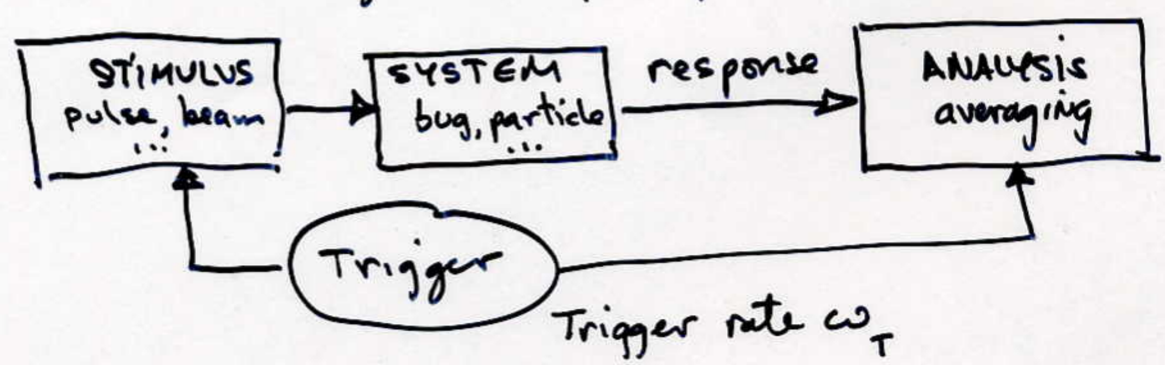
Making a signal periodic "CHOPPING"

Most measurements do not involve intrinsically periodic signals. We must force the signal to repeat. Signal depends on some external parameter, in general.

Vary the external parameter.

Examples:

- NMR (MRI) resonance ~ magnetic field. so vary the field.
- Mössbauer studies: vary the velocity
- Laser studies: chop the light beam.
- Response to stimulus: biophysics, condensed matter physics, high energy physics



Three methods of signal recovery

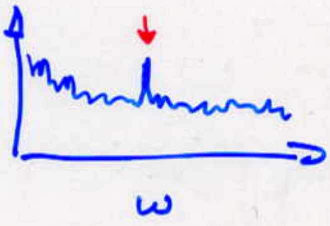
Direct detection

signal → [Detector] → output

works if signal >> noise.

Spectrum analyzer "Signal averaging"

signal → [Spectrum Analyzer] →



short averaging time

long averaging time:

works if signal $\sim e^{i\omega_s t}$!

Phase-Sensitive Detection

Modulate the signal at ω_0

Then detect in-phase component of signal + noise at ω_0.

Average signal in small bandwidth $\Delta f \sim \frac{1}{T}$ at f

T = averaging time.