CHAPTER

6

ELECTRONICS

This chapter discusses electronics at a level somewhere between that of a handbook, which consists essentially of charts, tables, and graphs, and a textbook, where the interesting, important, and useful conclusions come only after well-developed discussions with examples. The aim here is a presentation that has sufficient continuity and readability that individual sections can be profitably read without having to refer to preceding sections or other texts. On the other hand, it is important to have useful and frequently referenced material in the form of readily accessible tables, graphs, and diagrams that are sufficiently self-explanatory that very little reference to the text material is necessary. Another important goal is vocabulary. A large amount of jargon in electronics is meaningless to the uninitiated, but when it is necessary to understand the properties of an electronic device from a written technical description, when writing the specifications for electronic equipment, or when talking to an electronics engineer, salesman, or technician, this vocabulary is essential. With this in mind, terms not current outside of electronics are italicized.

To be used to best advantage, this chapter should be supplemented with manufacturers' catalogs, data books, applications texts, handbooks, and more specialized texts that treat the topic of interest in depth. Manufacturers of laboratory electronic equipment, discrete devices, and integrated circuits have publications that describe, in clear practical terms, the properties of their products and their applications to a wide variety of tasks. Much of this material is also available on the internet, and for this reason internet addresses are given when available.

The material has been organized and written as one explains it to a student or technician coming to work in a laboratory for the first time. The complexity of modern electronics is such that the cut-and-try approach is too inefficient and costly in terms of material and time. There are just too many possibilities when connecting devices and multiple-component circuits, and it is important to establish a systematic approach based on a limited number of simple, well-understood principles. It is probably not reasonable in the laboratory to expect quick solutions to problems that are entirely outside one's previous experience. The number of really new situations that can arise is limited, however, most problems being variations on a few basic situations. The ability to recognize this and to isolate the source of difficulty comes with practice and mastery of basic principles. When confronted with a new situation involving rack upon rack of equipment, the tendency is to believe that an understanding of how everything works is beyond the capabilities of all but expert electronics engineers. This is far from the truth. At the operational level, present-day electronics is the most reliable, easy-to-use, and easy-to-understand element of most experiments.

6.1 PRELIMINARIES

6.1.1 Circuit Theory

An understanding of elementary circuit theory and the accompanying vocabulary permits one to reduce complex circuits consisting of many elements to a few essentials, predict the behavior of complex circuits, specify the operation of components, and understand and use data sheets and operation manuals. In routine laboratory work, it is not necessary to be skillful with circuit theory. It is necessary to be able to isolate the basic elements of a circuit and understand their behavior. With that ability, when circuits fail to operate correctly, the causes of the malfunction can be localized and repaired.

Linear circuit theory applies to devices whose output is directly proportional to the applied input. If one increases the current through a resistor by a factor of two, for example, the voltage across it will double. An example of a nonlinear device is a switch that is either open or closed and whose state changes abruptly at a threshold. A nonlinear device can often be treated with linear theory by dividing the response of the device into separate regions over which it behaves in a quasi-linear manner. This is called *linearizing the response curve.* An example is the piecewise linearization of a diode's current–voltage response, as shown in Figure 6.1. The exponentially rising forward current and constant reverse current are represented by straight lines of slope $1/R_f$ and $1/R_r$, which are joined at voltage V_γ.

We begin by considering only *passive* linear devices; that is, devices that either dissipate energy (resistors) or store energy in electric (capacitors) or magnetic (inductors, transformers) fields. *Active* devices, such as transistors, can supply energy to a circuit when appropriately powered by external sources. The analysis of circuits with active devices is based on representations using equivalent circuits consisting of passive devices.

Conventional circuit analysis uses three *lumped* circuit elements – resistors (R), capacitors (C), and inductors (L). This way of analyzing circuits is valid at signal frequencies f for which the wavelength λ is much larger than the physical dimensions of the circuit. Since $\lambda = c/f$, where c is the speed of light, this means that analysis in terms of lumped elements is valid up to frequencies of a few hundred megahertz.

This frequency limitation also excludes waveforms with significant frequency components above a few hundred megahertz, even though the repetition rate of the waveform may be much less. A convenient way to estimate the frequency of the highest-frequency component of a nonsinusoidal waveform is to divide 0.3 by the *rise time* of the waveform, t_r, defined as the time between the 10% and 90% amplitude points on the leading edge of the waveform. A pulse with a 10 ns rise time, for example, has significant frequency components up to 30 MHz. The *fall*

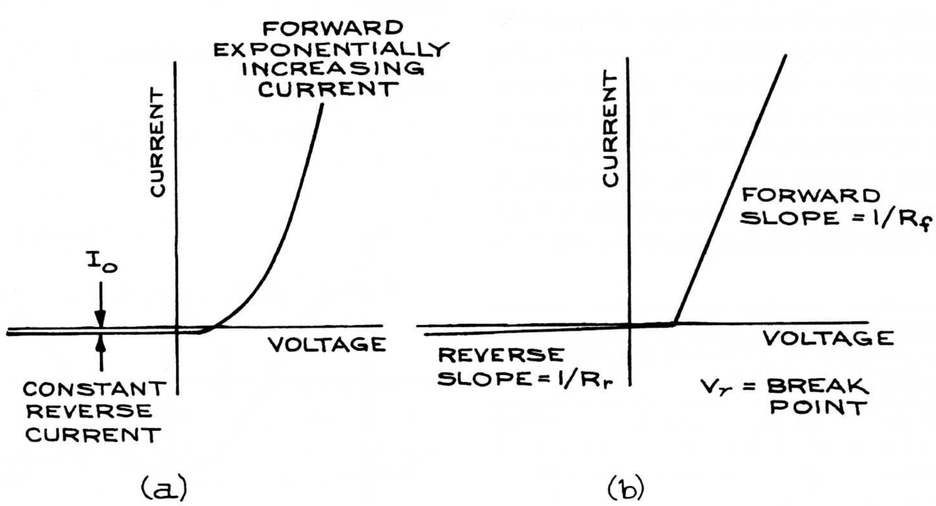


Figure 6.1 (a) Real and (b) piecewise linear representation of the current–voltage characteristics of a diode.

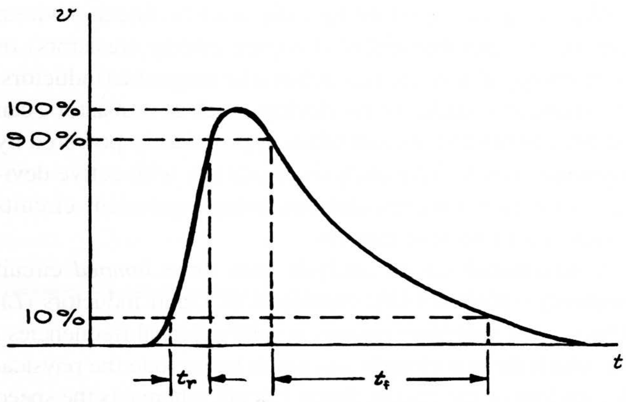


Figure 6.2 Rise time (t_r) and fall time (t_f) of a pulse.

time, t_f of a waveform is the time between the 10% and 90% amplitude points on the trailing edge. Figure 6.2 illustrates *rise time* and *fall time*.

Even at low frequencies there are no ideal resistors, capacitors, or inductors. Actual resistors have some capacitance and inductance, while capacitors have resistance and inductance, and inductors have resistance and capacitance. These departures from ideality are largely a matter of construction.

At high frequencies, stray capacitances and inductances become significant, and one commonly speaks of *distributed* parameters, in contrast to the lumped parameters at low frequencies. Coaxial cable is an example of a type of distributed parameter circuit. The electrical properties of coaxial cable are normally given in terms of resistance and attenuation per unit length as a function of frequency. At high frequencies, the resistance of conductors (even connecting wires) increases due to what is termed *skin effect.* The magnitude of this effect for round cross-section wires is given in Table 6.1 as a function of frequency. High-frequency connections are best made with leads having a large surface area-to-volume ratio, with flat-ribbon geometry the best.

Conventional circuit theory is based on a few laws, principles, and theorems. In the equations that follow, lowercase letters represent instantaneous values of voltage and current, whereas uppercase letters indicate effective or d.c. values. It is also convenient to distinguish between root-mean-square (rms), peak-to-peak, and average values of voltage and current for sinusoidally varying voltages. If $v = V\cos\omega t$, where v is the instantaneous value of voltage and V is the peak value, the rms value is $V/\sqrt{2}$, the peak-to-peak value is $2V$, and the average value is clearly zero. This is illustrated in Figure 6.3. Common US line voltage is specified as 110 V a.c., which is the rms value. The peak voltage is 156 V, so the peak-to-peak voltage is 312 V.

Under certain conditions, the rms value is not sufficient for specifying the output of an a.c. source. A source producing voltage spikes of large amplitude, but short duration superimposed on a small sinusoidally varying voltage will have an rms value very close to that without the spikes, but the spikes can have a large effect on circuits connected to the source. When specifying the output of a d.c. power supply, the magnitude, frequency, and duration of nonsinusoidal waveforms that appear at the output need to be specified, as well as the rms value of any a.c. component of the output.

Table 6.1 Ratio of a.c to d.c wire resistance

Wire Gauge	R_{ac}/R_{dc}			
	10^6 *Hz*	10^7 *Hz*	10^8 *Hz*	10^9 *Hz*
#22	6.9	21.7	69	217
#18	10.9	34.5	109	345
#14	17.6	55.7	176	557
#10	27.6	87.3	276	873

Laws

(i) Current–voltage relations. For resistors, capacitors, and inductors we have:

$$v_R = iR, \quad v_C = \frac{1}{C}\int i\,\mathrm{d}t, \quad v_L = L\frac{\mathrm{d}i}{\mathrm{d}t} \tag{6.1}$$

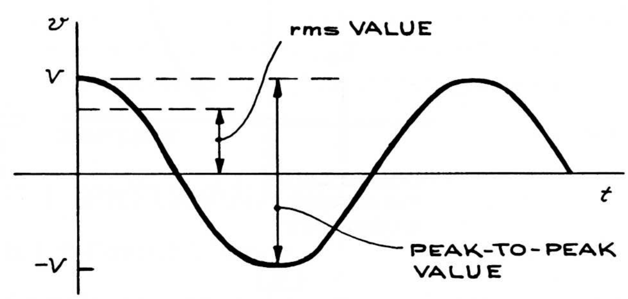


Figure 6.3 Relation of rms and peak-to-peak voltages for a sinusoidal waveform.

respectively, where the resistance R is in ohms; the capacitace C in farads; and the inductance L in henrys.

(ii) Loops and nodes (Kirchhoff's laws). In these, the sums are algebraic (signs taken into account):

(1) Σ (voltage drops around a closed loop) $= \Sigma$ (voltage sources).

(2) Σ (current into a node) $= \Sigma$ (current out of a node), where a node is a point where two or more elements have a common connection.

Theorems

(i) Thevenin's theorem. A real voltage source in a circuit can always be replaced by an ideal voltage source in series with a generalized resistance. An ideal voltage source is one that can maintain a constant voltage across its terminals regardless of the load across it. In other words, an ideal voltage source has zero internal resistance. An automobile battery, with an internal resistance of a few hundredths of an ohm, is a good approximation to an ideal voltage source at currents of a few amperes. Electronically regulated power supplies often have very low effective internal resistances when operated within their voltage and current ratings.

(ii) Norton's theorem. A real current source in a circuit can always be replaced by an ideal current source shunted by a generalized resistance. An ideal current source is one that supplies a constant current regardless of load – such a source has an infinite internal resistance. Photo-multiplier and electron-multiplier devices provide currents, albeit very low, that are almost independent of load, and they approximate ideal current sources.

Superposition, Circuits with Multiple Sources

For a circuit that contains several sources (voltage, current, or a combination of both) the contribution of each source to the voltage between any two points or the current past a point can be considered separately with all the other sources represented by their internal resistances. The total voltage or total current is then the algebraic sum of the separate contributions of each of the individual sources.

When connecting a source of current or voltage to a circuit, it is often important to know the internal resistance of the source. This can be determined by first measuring the open-circuit voltage of the source with a high internal-resistance voltmeter and then connecting a variable resistance across the source and adjusting it until the voltmeter reading is one half the open-circuit value. The source resistance is then equal to the value of the variable-resistance setting. If the source has a very high internal resistance, a current measurement can be substituted for the voltage measurement. In this case, the output is shunted with an ammeter and the so-called *short-circuit current* is measured. A variable resistance is then placed in series with the ammeter and adjusted until the current through the ammeter is one half the short-circuit current. The value of the variable resistor at this point is equal to the internal resistance of the source. Analogous measurements can be made for a.c. sources by using either a.c. voltmeters and ammeters or an oscilloscope.

6.1.2 Circuit Analysis

For any given source, the choice of representation (Thevenin or Norton) is arbitrary and, in fact, the series resistance in the Thevenin representation is exactly equal to the parallel resistance in the Norton representation. Thevenin's and Norton's theorems simplify the application of the laws and principles discussed above.

The most general method for solving circuit problems is to apply Kirchhoff's laws using the appropriate current–voltage relations for each element in the circuit. This gives rise to one or more linear differential equations, which, when solved with the proper boundary conditions, give the general solution. This is illustrated for RC circuits in Section 6.1.3.

When dealing with sinusoidal sources of angular frequency ω, circuit analysis can be greatly simplified when only the steady-state solution is required. In this case, circuit capacitances and inductances are replaced by *reactances:*

$$\begin{aligned}
&\text{capacitive reactance} = jX_C, \quad \textit{where } X_C = \frac{-1}{\omega C} \\
&\text{inductive reactance} = jX_L, \quad \textit{where } X_L = \omega L \qquad (6.2) \\
&j = \sqrt{-1}
\end{aligned}$$

The *impedance* Z of a circuit is obtained by combining reactances and resistances according to the formula $Z = R + j(X_L + X_C)$. These quantities can be represented in the complex plane by vectors (see Figure 6.4). The angle between Z and the real axis is ϕ. By analogy with the *I–V* relations for a pure resistance, X_C is the ratio of the a.c. voltage across a capacitor to the current through it, X_L is the ratio of the a.c. voltage across an inductor to the current through it, and Z is the net ratio of a.c. voltage to current in a circuit composed of resistors, capacitors, and inductors.

The fact that jX_C and jX_L are imaginary means that the voltage and current are 90° out of phase with each other. For a capacitor, the voltage lags the current by 90°, while for an inductor the voltage leads the current by 90°.

Another quantity that is occasionally useful in circuit analysis is the *complex admittance Y*, which is the reciprocal of the impedance. The SI unit of admittance is the *siemen*. The usefulness of the admittance arises in circuits with several parallel branches, where the net admittance is the sum of the admittances of the branches.

In carrying out circuit analysis, the following results of the above laws are useful:

Series Circuits. At any instant the current is the same everywhere in a series circuit, and the algebraic sum of the voltage drops around a circuit equals the algebraic sum of the sources. For circuit elements of impedance $Z_1, Z_2, \ldots, Z_N$ in an N element series circuit, the total impedance is $Z = Z_1 + Z_2 + \cdots + Z_N$. If all the elements are resistors, or inductors, or capacitors, the general expression reduces, respectively, to:

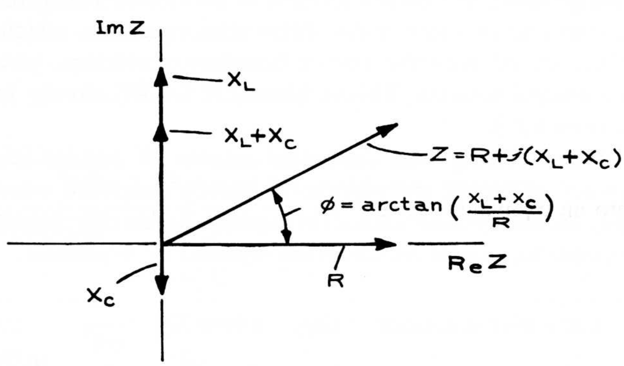


Figure 6.4 Relations between reactance, resistance, impedance, and phase angle.

$$R = R_1 + R_2 + \cdots + R_N$$
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N} \tag{6.3}$$
$$L = L_1 + L_2 + \cdots + L_N$$

Parallel Circuits. For circuit elements in parallel, the voltage drop across each branch is the same while the current through each branch is inversely proportional to the impedance of the branch. The total current through all of the branches is the voltage across the network divided by the equivalent impedance for the network. The equivalent impedance Z and admittance Y for an N branch parallel circuit are:

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_N} \tag{6.4}$$

and:

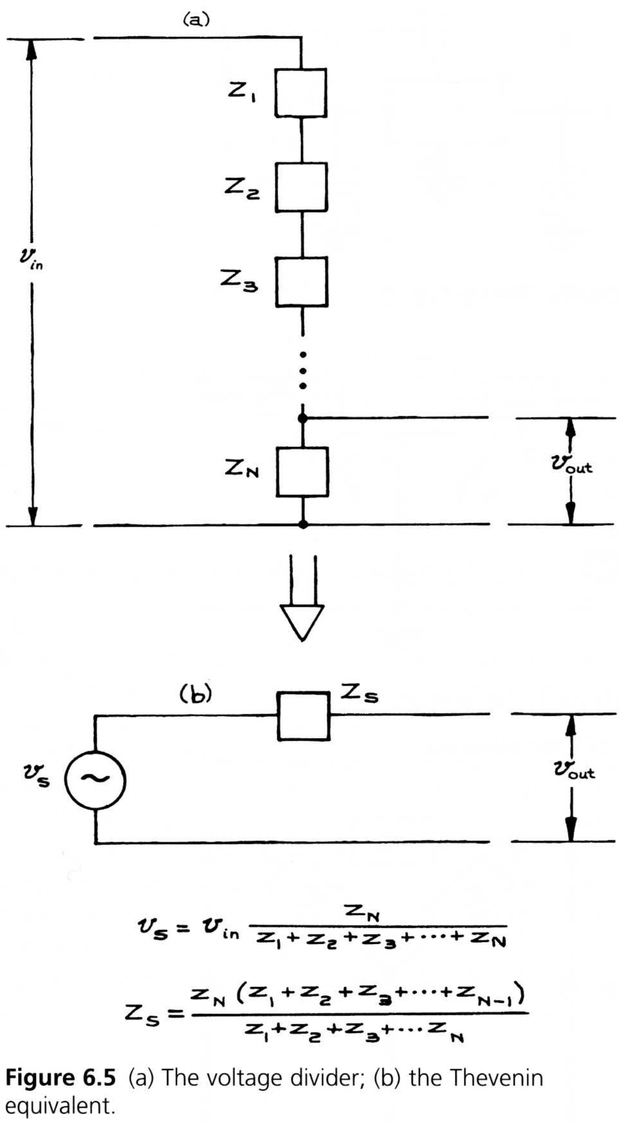
$$Y = Y_1 + Y_2 + \cdots + Y_N \tag{6.5}$$

where $Z_1, Z_2, \ldots, Z_N$ are the impedances of the branches and $Y_1, Y_2, \ldots, Y_N$ are the admittances. In the special cases where all the circuit elements in the branches are of the same type:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$
$$C = C_1 + C_2 + \cdots + C_N \tag{6.6}$$
$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}$$

where R, C, and L are the net resistance, capacitance, and inductance of the circuits.

Voltage Dividers. The *voltage divider*, illustrated in Figure 6.5(a), is a very common circuit element. The instantaneous voltage across Z_N is $v_{in}[Z_N/(Z_1 + Z_2 + Z_3 + \cdots + Z_N)]$; that is, the fraction of v_{in} that appears



$$v_s = v_{in}\,\frac{Z_N}{Z_1 + Z_2 + Z_3 + \cdots + Z_N}$$

$$Z_s = \frac{Z_N\,(Z_1 + Z_2 + Z_3 + \cdots + Z_{N-1})}{Z_1 + Z_2 + Z_3 + \cdots Z_N}$$

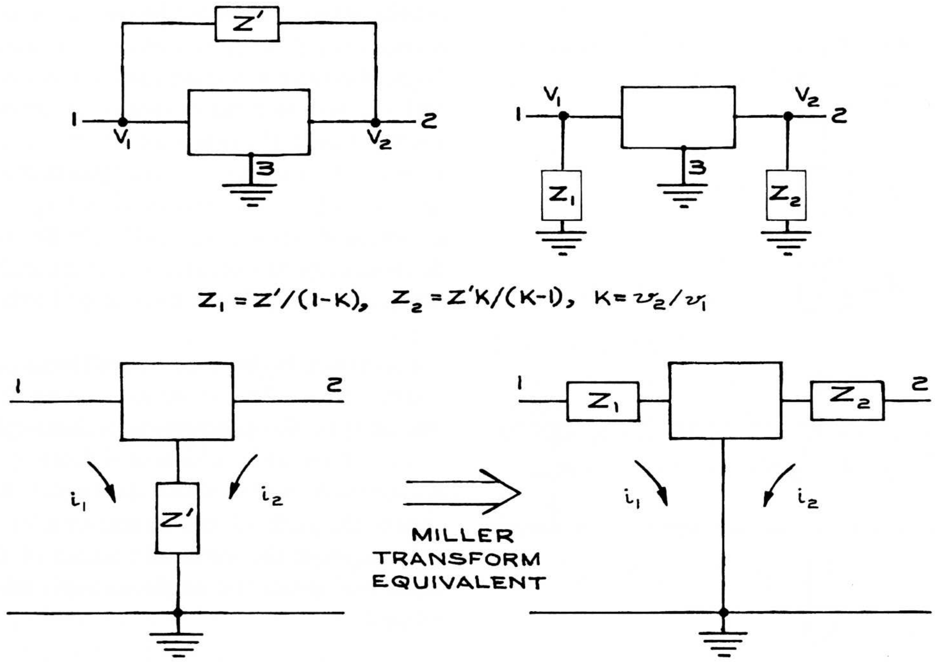
Figure 6.5 (a) The voltage divider; (b) the Thevenin equivalent.

across any circuit element is the impedance of that element divided by the total impedance of the series circuit. Voltage dividers provide a convenient way to obtain a variable-voltage output from a fixed-voltage input, but there are limitations. To avoid drawing too much current from the voltage source, the impedance of the voltage divider string should not be too small. If, in the interest of conserving power, however, the impedance is made large, the output impedance of the circuit will be large and v_{out} will depend critically on the load. This can be seen from the Thevenin equivalent of the circuit given in Figure 6.5(b) where v_s is the instantaneous voltage of the ideal voltage source. When Z_s is large, the voltage across a load will depend critically on the value of the load. Such *loading* of a divider is to be avoided. For noncritical applications, Z_s should be at most 1/10 of any anticipated load.

Precision, highly linear, multiturn potentiometers are commonly used for position sensing. In this application, the shaft of the potentiometer is coupled mechanically to the moveable element whose position is to be determined, and a stable voltage source is connected across the ends of the potentiometer. The ratio of the voltage from the variable contact of the potentiometer to its end gives the angle through which the shaft has rotated.

Equivalent Circuits. Two circuits are *equivalent* if the relationships between the measurable currents and voltages are identical. As has been seen, a circuit with two external terminals can be replaced by its Thevenin or Norton equivalent. Common equivalence transformations for circuits with three terminals (*Miller* and *Y–Δ transformations*) are shown in Figures 6.6 and 6.7. In the first Miller transformation of Figure 6.6, it is necessary to know the ratio of the voltages at nodes 1 and 2; in the second it is necessary to know the ratio of the currents into nodes 1 and 2.

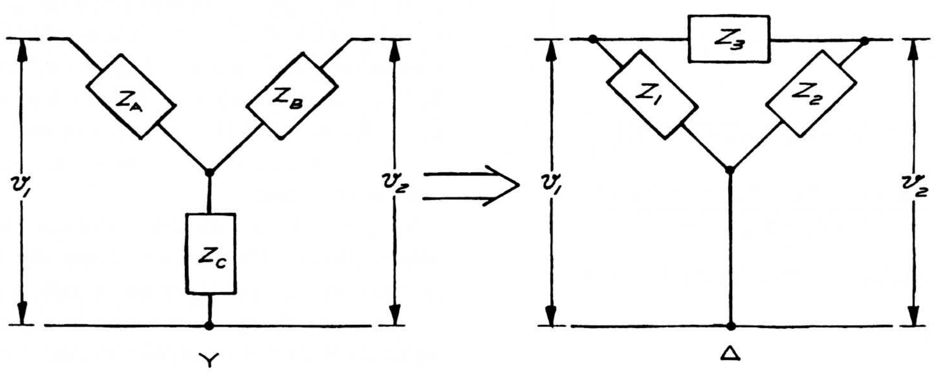
Discussions of amplifier circuits often refer to the *Miller effect.* This occurs when the input and output circuits of the amplifier are coupled by an impedance Z′. By using the transformation in Figure 6.6, Z′ between input terminal 1 and output terminal 2 can be transformed into an equivalent circuit where Z′ is replaced by Z_1 and Z_2 from terminals 1 and 2 to ground. The relations between Z_1, Z_2, and Z′ are given in the figure. When the coupling between terminals 1 and 2 is capacitive with $Z' = 1/j\omega C'$, Z_1 and Z_2 are also capacitive impedances equal to $1/j\omega C'(1-K)$ and $1/j\omega C'K/(K-1)$, where K is the voltage gain of the amplifier, negative for the example shown in Figure. 6.6.



$$Z_1 = Z'/(1-K), \quad Z_2 = Z'K/(K-1), \quad K = v_2/v_1$$

$$Z_1 = Z'(1-A_i), \quad Z_2 = Z'(A_i-1)/A_i, \quad A_i = -i_2/i_1$$

Figure 6.6 Miller transformations for circuits with three terminals.



$$Z_1 = \frac{Z_A Z_B + Z_A Z_C + Z_B Z_C}{Z_B}$$

$$Z_2 = \frac{Z_A Z_B + Z_A Z_C + Z_B Z_C}{Z_A}$$

$$Z_3 = \frac{Z_A Z_B + Z_A Z_C + Z_B Z_C}{Z_C}$$

Figure 6.7 Y–Δ transformation for a circuit with three terminals.

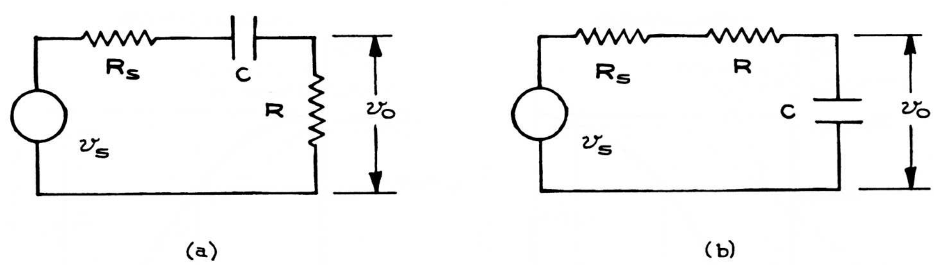


Figure 6.8 (a) High-pass or differentiating circuit; (b) low-pass or integrating circuit.

The Y–Δ transformation allows one to transform a circuit of three elements from a node to a loop configuration.

6.1.3 High-Pass and Low-Pass Circuits

Analysis of the *high-pass* and *low-pass* circuits shown in Figure 6.8 illustrates some of the above circuit-analysis principles. The combination of v_s and R_s represents a real voltage source with instantaneous open-circuit voltage v_s and internal resistance R_s. For the high-pass or *differentiating circuit,* the output voltage v_o is across the resistor R; for the low-pass or *integrating circuit,* it is across the capacitor C. Very often, the essential properties of complex circuits can be understood in terms of one of these two circuits, so it is useful to be acquainted with their characteristics.

These circuits can be analyzed by the *differential-equation method.* For either circuit:

$$v_s(t) = iR_s + \frac{1}{C}\int i\,dt + iR \tag{6.7}$$

This equation uses the fact that the sum of the voltage drops in the circuit equals the sum of the voltage sources and the current is the same everywhere in a series circuit at any instant. Differentiating with respect to time:

$$\frac{dv_s}{dt} = \frac{di}{dt}(R_s + R) + \frac{i}{C} \tag{6.8}$$

The solution to the homogeneous equation ($dv_s/dt = 0$) is:

$$i = A\,e^{-t/R'C} \tag{6.9}$$

where $R' = R_s + R$ and A is the integration constant determined from the initial conditions. The general solution requires that the functional form of v_s be known. Consider three cases:

(1) An a.c. voltage of amplitude V:

$$v_s = V\cos(\omega t + \phi) \tag{6.10}$$

(2) A step voltage of amplitude V:

$$v_s = \begin{cases} 0 \text{ for } t < 0 \\ V \text{ for } t > 0 \end{cases} \tag{6.11}$$

(3) A rectangular pulse of amplitude V and duration T:

$$v_s = \begin{cases} V \text{ for } 0 \leqslant t \leqslant T \\ V \text{ for } t < 0,\ t > T \end{cases} \tag{6.12}$$

For case 1, the output voltage is sinusoidal at the same frequency as the input voltage. The ratio of v_o to v_s as a function of normalized frequency is shown in Figure 6.9(a). At the frequencies $\omega = \omega_H$ and $\omega = \omega_L$ for the two circuits, v_o is $1/\sqrt{2}$ of the maximum value. These frequencies are called the *upper* and *lower corner frequencies,* respectively.

The maximum power that can be delivered to a load is proportional to the square of the output voltage, so that at $\omega = \omega_H$ and $\omega = \omega_L$, the maximum power that the circuits

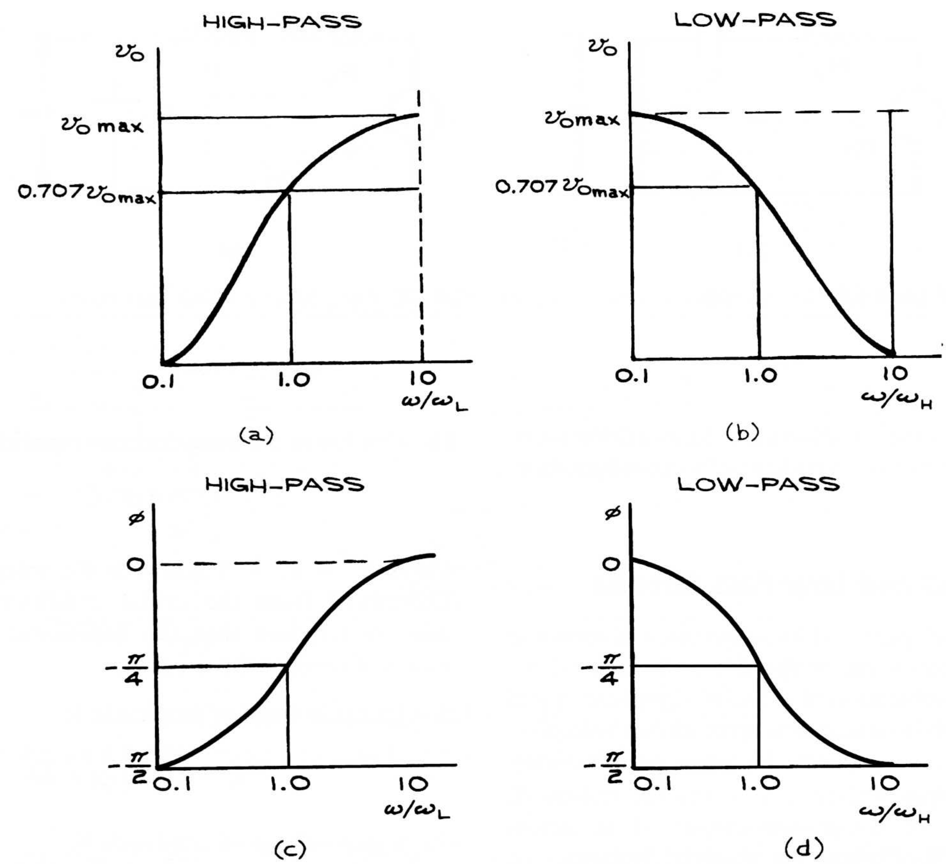


Figure 6.9 Output voltage as a function of frequency for: (a) high-pass and (b) low-pass circuits; phase as a function of frequency for: (c) high-pass and (d) low-pass circuits.

can deliver to a constant load is one-half the maximum possible value. The usual way of expressing this is in terms of *decibels* dB, where:

$$\begin{aligned}\text{ratio in dB} &= 10\log_{10}\left(\frac{\text{power out}}{\text{power in}}\right)\\ &= 10\log_{10}\left(\frac{v_{\text{out}}^2/R_{\text{out}}}{v_{\text{in}}^2/R_{\text{in}}}\right)\end{aligned} \tag{6.13}$$

If $R_{\text{out}} = R_{\text{in}}$, which is often assumed, then (ratio in dB) = 20 $\log_{10}[v_{\text{out}}/v_{\text{in}}]$. When $v_{\text{out}}/v_{\text{in}}=\sqrt{1/2}$, this is approximately -3, so that -3 dB represents a power reduction of a factor of two. Since the frequency response of amplifiers, filters, and transducers is routinely given in dB, it is important to keep in mind that the dB scale is logarithmic. Human sensory perception is approximately logarithmic, and a 3 dB change in sound level or light level is barely perceptible.

Because of the reactive element in the *RC* circuits (the capacitor), the voltage is not in phase with the current, as illustrated in Figure 6.9(b). These plots of phase and log (output voltage) as a function of log(frequency) are called *Bode plots,* after H. W. Bode.[1]

It is often convenient to approximate frequency-response curves by a piecewise linear function. Such idealized Bode plots are shown in Figure 6.10(a) and (b). The *corner frequencies* are where ω/ω_L and ω/ω_H = 1.0.

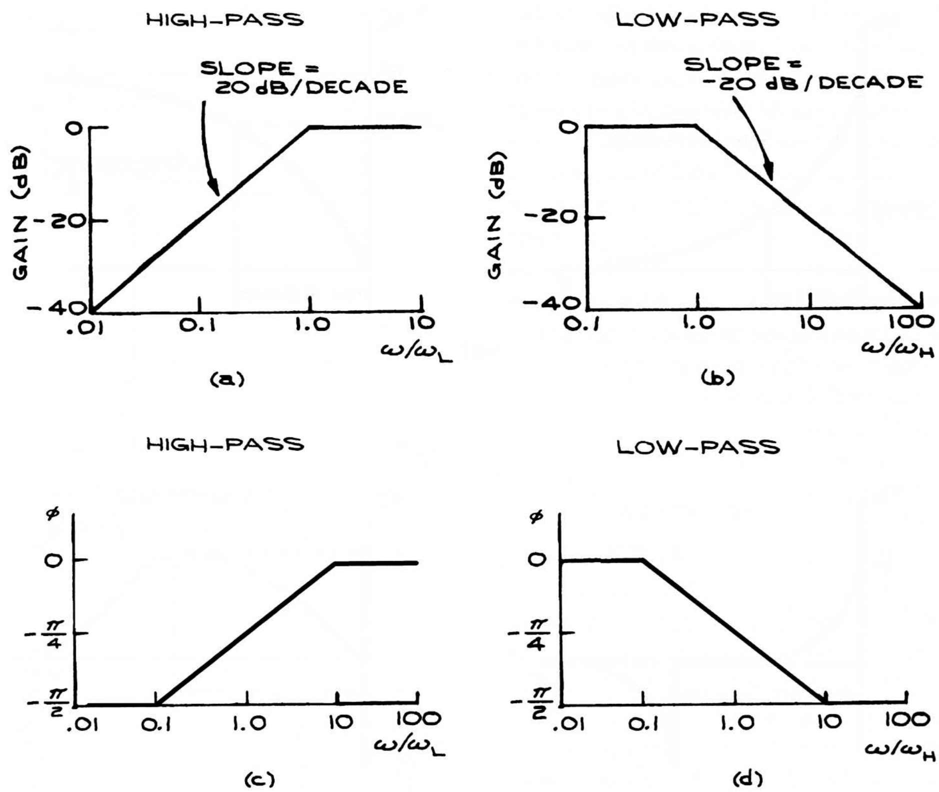


Figure 6.10 Idealized gain response for (a) high-pass and (b) low-pass circuits; idealized phase response for (c) high-pass and (d) low-pass circuits

They correspond to the -3 dB points on the unapproximated Bode plots. For most purposes, the simplified curves are satisfactory representations. From these curves, every 10-fold reduction in frequency below ω_L for the high-pass circuit decreases the output voltage by 20 dB, and every twofold reduction decreases it by 6 dB. The low-pass circuit has just the opposite properties: a 10-fold increase in frequency above ω_H results in a 20 dB decrease in output voltage, and a twofold increase results in a 6 dB decrease. One often states these facts as *20 dB per decade* and *6 dB per octave*. The linearized phase-response curves are shown in Figure 6.10(c). The -3 dB frequencies occur at a phase shift of $-\pi/4$ ($-45°$) for the two circuits.

For the nonrepetitive input voltage waveforms of cases (2) and (3), the output waveforms are given in Figure 6.11.

The output, waveforms for the rectangular-wave input function can be used to determine the RC time constants for differentiating and integrating circuits. This is called *square-wave testing*. The RC time constant for the differentiating circuit is obtained by using a square-wave input with a rise time much smaller than RC and a period much larger than RC. For times small compared with RC, the tilt of the top edge of the output, as viewed with a fast-rise-time oscilloscope (see Figure 6.12) is directly related to RC. The fractional decrease in v_o, in time t_1 is t_1/RC, which can be set equal to $(V - V')/V$ and solved for RC. For the integrating circuit, RC is obtained by measuring the rise time of the output waveform on a fast-rise-time oscilloscope. Using the definition of the rise time t_r, as the time between the 10% and 90% points

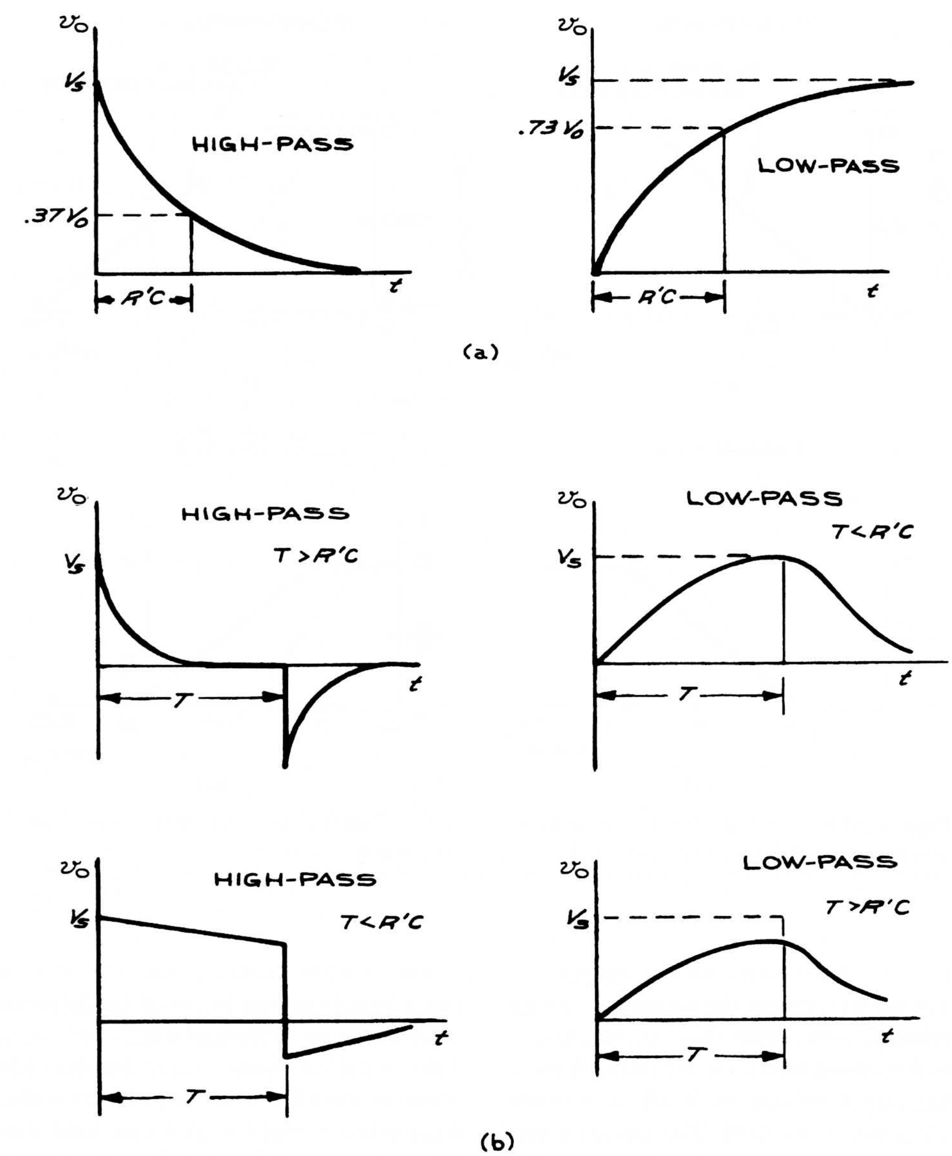


Figure 6.11 Response of high-pass and low-pass circuits to a step voltage (a) and a rectangular pulse of duration T (b).

on the leading edge of the output waveform, one has the relation:

$$t_r = 2.2RC \tag{6.14}$$

6.1.4 Resonant Circuits

The voltages and currents in circuits with capacitors, inductors, and resistors show oscillatory properties much like those of mechanical oscillators. Electronic circuits have

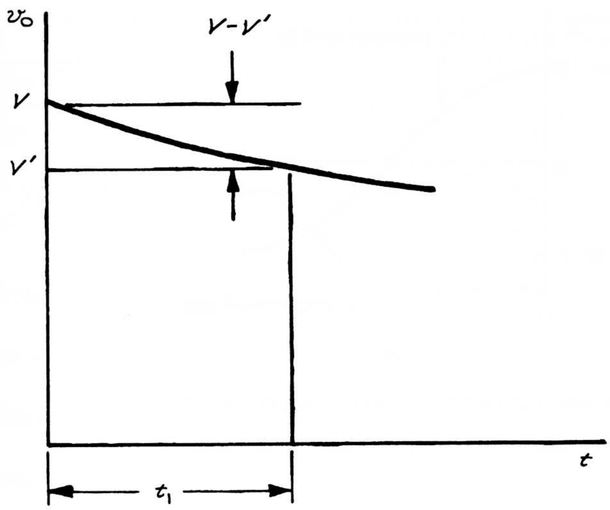


Figure 6.12 Square-wave testing of a high-pass circuit.

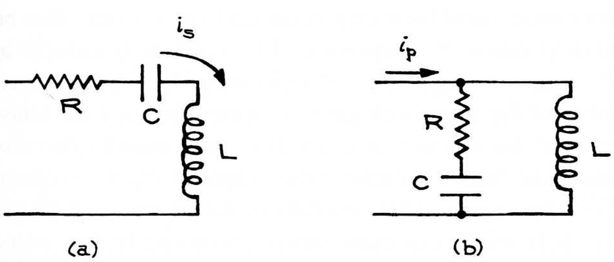


Figure 6.13 (a) Series resonant circuit; (b) parallel resonant circuit.

natural frequencies of oscillation and can be critically damped, underdamped, or overdamped, depending on the relations between the values of the circuit parameters. Resonant circuits with ideal capacitors and inductors are of the series or parallel type shown in Figure 6.13. When driven by a sinusoidal input source, the capacitative reactance in the series circuit will cancel the inductive reactance at the resonant frequency ω_o, where $1/C\omega_o = \omega_o L$ and $\omega_o = \sqrt{1/LC}$. At ω_o the impedance of the series circuit is a minimum and the current through it is a maximum.

For the parallel resonant circuit at low frequencies, the L branch will have a very low reactance and the current drawn from the source will flow almost entirely through that branch. At high frequencies, the current through the RC branch is limited by the value of R. The total impedance of the parallel circuit is therefore small at low and high frequencies, passing through a maximum at the frequency $\omega_o = \sqrt{1/LC}$, provided that $R << \omega_o L$. Graphs of the currents in the two circuits as a function of driving frequency are given in Figure 6.14. Real inductors have an associated resistance, which generally must also be taken into account when analyzing circuits.

One measure of the resonance sharpness in the series and parallel circuits is the Q or *quality* of the circuit. For practical purposes, $Q = \omega_o/\Delta\omega$ where $\Delta\omega$ is the full width at half maximum of the peak or valley. In terms of the circuit parameters, $1/Q = \omega_o L/R = 1/\omega_o RC$. This is the ratio of the energy stored (in the capacitor or inductor) to the energy dissipated in the resistor per cycle at resonance. Values of Q

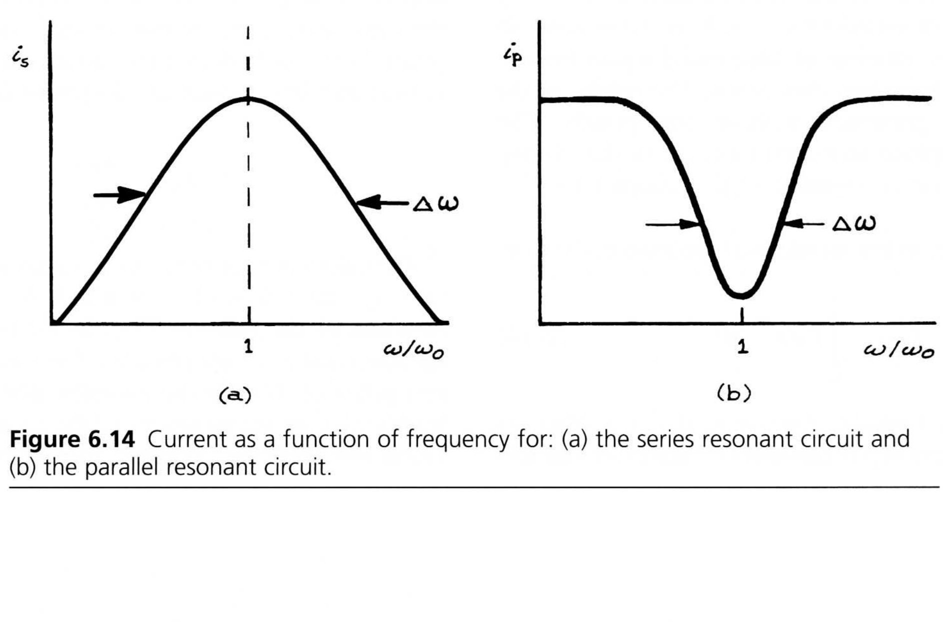


Figure 6.14 Current as a function of frequency for: (a) the series resonant circuit and (b) the parallel resonant circuit.

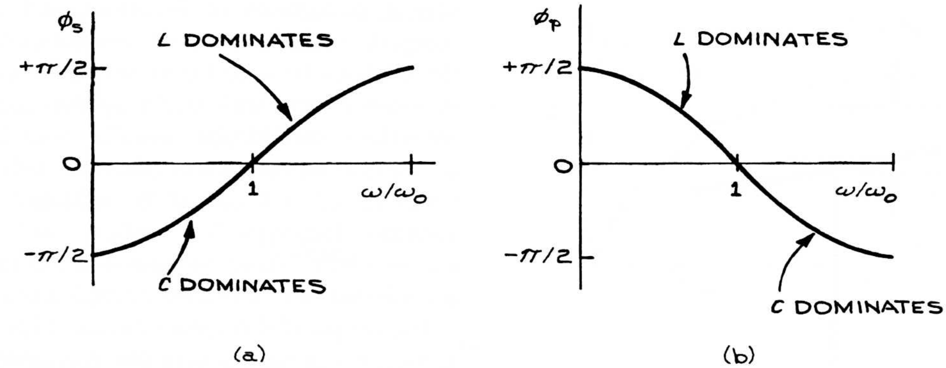


Figure 6.15 Phase relations between voltage and current in: (a) the series resonant circuit and (b) the parallel resonant circuit.

as large as 100 can be attained in electrical circuits while mechanical oscillators can attain values as high as 10^6. The phase relationships between voltage and current in series and parallel resonant circuits are shown in Figure 6.15.

The behavior of an *LRC* circuit upon the application of a step or rectangular input is much like the response of a mechanical system to a sudden impulse. Critically damped, underdamped, and overdamped current flows result.

6.1.5 The Laplace-Transform Method

A general technique for analyzing circuits for arbitrary input voltage waveforms is the method of Laplace transforms. With this method it is possible to use only algebra and lists of transforms – such as those given in Table 6.2 – for the solution of differential equations and the evaluation of boundary conditions. The results of the method will be presented without any proofs. The vocabulary of Laplace transforms occurs in the discussion of circuits and is included in this chapter for that reason.

The method is based on an integral transform of the type:

$$\bar{f}(s) = \int_0^\infty f(t)\mathrm{e}^{-st}\mathrm{d}t \qquad (6.14)$$

where $\bar{f}(s)$ is the Laplace transform of *f(t)*, written as $\mathscr{L}$ [*f*(*t*)]. The function *f(t)* can involve integrals and differentials. When $\mathscr{L}$ is applied to the second-order differential equations that arise in circuit analysis, rather important simplifications occur and results can often be written down by inspection. The Laplace transform of the output voltage $\bar{v}_\mathrm{o}(s)$ of a circuit is the Laplace transform of the input voltage $\bar{v}_i(s)$ times the Laplace transform of the transfer function $\bar{T}(s)$ – the *transfer function* being the function relating the output to input. To obtain $\bar{T}(s)$, the values of all elements in the circuit are replaced by their transform equivalents according to the recipe $R \rightarrow R$, $C \rightarrow 1/sC$, and $L \rightarrow sL$. In the simple case of the voltage divider, the transfer function is the ratio of the impedance of the output-circuit element to the total impedance of the circuit chain. *T*(*s*) is obtained in exactly the same way, using the equivalences for *R*, *C*, and *L*. In general, $\bar{T}(s)$ is in the form of a ratio of two functions of *s*, *G*(*s*) and *H*(*s*), which are polynomials in *s*:

$$\bar{T}(s) = \frac{G(s)}{H(s)} \qquad (6.15)$$

The values of *s* for which *G*(*s*) is zero are called the *zeros* of $\bar{T}(s)$; the values of *s* for which *H*(*s*) is zero are the locations of the *poles* of $\bar{T}(s)$. In the most general case, the zeros and poles are complex. The positions of the zeros and poles of $\bar{T}(s)$ in the complex plane give important information on the properties of the circuit under analysis. When the poles are complex, they occur in pairs, while

Table 6.2 Elementary Laplace Transforms

$f(t)(t > 0)$	$\bar{f}(s)$
$\delta(t)$	1
1	$1/s$
$t^{n-1}/(n-1)!$	$1/s^n$(n a positive integer)
e^{at}	$\dfrac{1}{s-a}$
$\sin at$	$\dfrac{a}{s^2+a^2}$
$\cos at$	$\dfrac{s}{s^2+a^2}$
$\sinh at$	$\dfrac{s}{s^2-a^2}$
$\cosh at$	$\dfrac{s}{s^2-a^2}$
$\dfrac{t}{2a}\sin at$	$\dfrac{s}{(s^2+a^2)^2}$
$\dfrac{1}{2a^3}(\sin at - at\cos at)$	$\dfrac{1}{(s^2+a^2)^2}$
$\dfrac{df(t)}{dt}$	$s\bar{f}(s) - f_0, f_0 = \lim_{t\to 0} f(t)$
$\dfrac{d^2f(t)}{dt}$	$s^2\bar{f}(s) - sf_0 - f_1, f_1 = \lim_{t\to 0}\dfrac{df(t)}{dt}$
$\int_0^t f(t')dt'$	$\dfrac{1}{s}\bar{f}(s)$
$\dfrac{1}{a-b}(e^{at} - e^{bt})$	$\dfrac{1}{(s-a)(s-b)}$
$\dfrac{1}{a-b}(ae^{at} - be^{bt})$	$\dfrac{s}{(s-a)(s-b)}$
$\dfrac{1}{a^2}(1-\cos at)$	$\dfrac{1}{s(s^2+a^2)}$
$\dfrac{1}{a^3}(at - \sin at)$	$\dfrac{1}{s^2(s^2+a^2)}$
$\dfrac{1}{ab(b^2-a^2)}(b\sin at - a\sin bt)$	$\dfrac{1}{(s^2+a^2)(s^2+b^2)}$
$\dfrac{1}{b^2-a^2}(\cos at - \cos bt)$	$\dfrac{s}{(s^2+a^2)(s^2+b^2)}$
$\dfrac{1}{\alpha^2+\beta^2} - \dfrac{e^{-\alpha t}}{\beta\sqrt{\alpha^2+b^2}}\sin[\beta t + \arg(\alpha + i\beta)]$	$\dfrac{1}{s\left[(s+\alpha)^2+\beta^2\right]}$

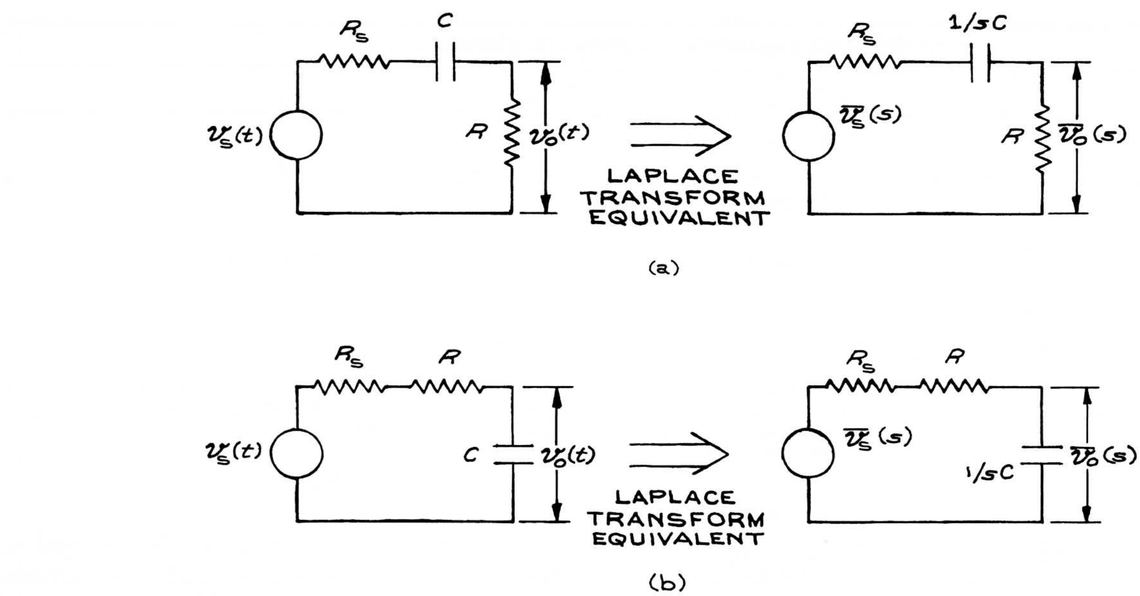


Figure 6.16 Laplace-transform equivalents of: (a) the high-pass and (b) the low-pass circuit.

real-valued poles can occur singly or in pairs. The values of the real and imaginary components of the pole coordinates, usually labeled σ and ω, have important physical meaning. The real component σ is a measure of the damping in the circuit while the imaginary part ω is the natural frequency of oscillation. Negative values of σ give stable circuits in which transient signals all decay to zero with time. Circuits employing only passive elements behave in this way and are stable. Circuits with active elements can behave in such a way that the output increases with time in response to a transient input signal. Such circuits are unstable and have values of σ greater than zero. They are to be avoided, except in the case of oscillators, which must be unstable in order to function.

The Laplace-transform equivalents of the high-pass and low-pass circuits are shown in Figure 6.16. For the high-pass circuit, the output voltage across the resistor R for the transformed circuit is:

$$\bar{v}_o(s) = \bar{v}_s(s)\frac{1}{(1+\frac{R_s}{R})+\frac{1}{SRC}} \tag{6.16}$$

where $\bar{T}(s)$, the transfer function, has a pole at $s = -1/(R + R_s)C$. For the low-pass circuit, the output voltage across the capacitor for the transformed circuit is:

$$\bar{v}_o(s) = \bar{v}_s(s)\frac{1}{1+(1+\frac{R_s}{R})SRC} \tag{6.17}$$

$\bar{T}(s)$ has a pole at $S = -1/(R+R_s)C$. The steady-state frequency and phase response of the circuits are obtained from $\bar{T}(s)$ by replacing s with $j\omega$. The transfer functions are now, for the high-pass circuit:

$$T(\omega) = \frac{1}{(1+\frac{R_s}{R})-j\frac{\omega_L}{\omega}} \tag{6.18}$$

and:

$$T(\omega) = \frac{1}{1+j\frac{\omega}{\omega_H}(1+\frac{R_s}{R})} \tag{6.19}$$

for the low-pass circuit. For these circuits $\omega_L = \omega_H = 1/RC$ for $R_s|R<<|$.

As seen previously, ω_L and ω_H are the corner frequencies of the circuits. By rationalizing the denominators of the transfer functions, one obtains the phase response. Since there are no inductive elements in the circuits, there is no natural frequency of oscillation. The poles lie on the negative real axis because there are no active elements in the circuit to cause sustained oscillations.

6.1.6 *RLC* Circuits

Consider the equivalent circuit and the Laplace transform given in Figure 6.17. To analyze the circuit, consider the parallel combination of R and $1/sC$, which is in series with sL in a voltage-divider configuration:

$$\frac{R/sC}{R+1/sC}=\frac{R}{1+sRC} \tag{6.20}$$

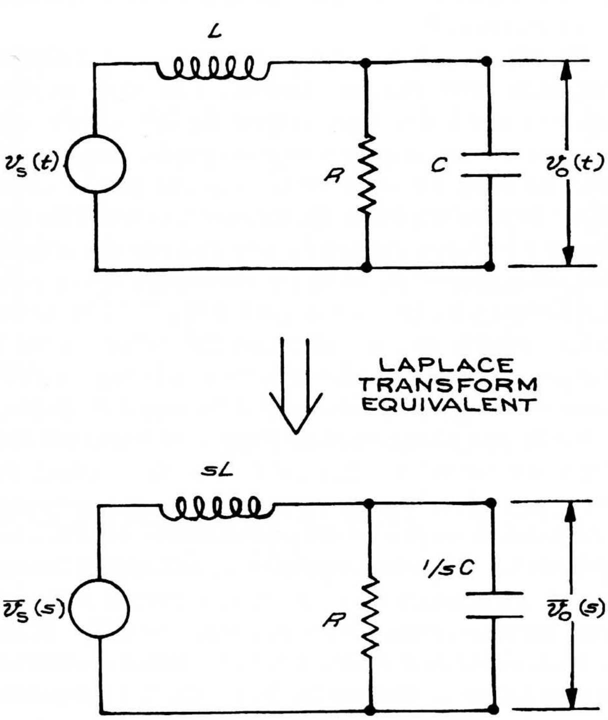


Figure 6.17 An *RLC* circuit and the Laplace-transform equivalent.

Thus:

$$\bar{v}_o(s)=\bar{v}_i(s)\frac{1}{1+sL/R+s^2LC} \tag{6.21}$$

The poles of $\bar{T}(s)$ occur at:

$$s=\frac{-L/R\pm\sqrt{(L/R)^2-4LC}}{2LC} \tag{6.22}$$

Letting the natural frequency of oscillation of the circuit be $\omega_0=1/\sqrt{LC}$, we have $Q=R/\omega_0L_0$, and the poles can be rewritten as:

$$s=\frac{-\omega_0}{2Q}\pm\frac{\omega_0}{2}\sqrt{1/Q^2-4} \tag{6.23}$$

There are three different possibilities for the roots of s:

(1) $1/Q^2=4$: a single real root at $s=-\omega_0/2Q$
(2) $1/Q^2-4=m^2$ (m real): two real roots at $s=-\omega_0/2Q \pm \omega_0 m/2$.
(3) $1/Q^2-4=-m^2$ (m real): two conjugate complex roots at $s=-\,\omega_0/2Q\pm j\omega_0 m/2$.

The magnitude of s in the $\sigma, j\omega$ plane is ω_0; in geometric terms this means that the roots of s are confined to a semicircle of radius ω_0 in the left half of the complex plane (see Figure 6.18).

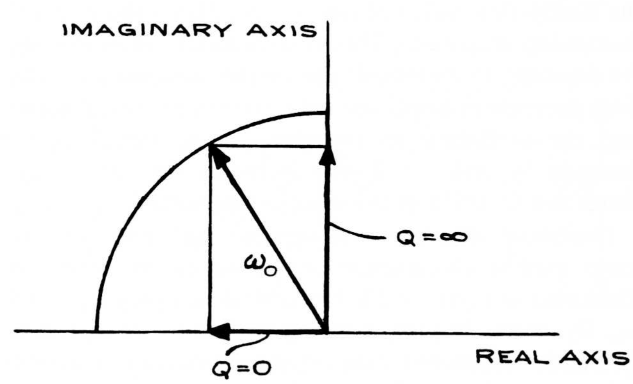


Figure 6.18 Pole trajectory for an *RLC* circuit.