The work being carried on at present in this laboratory is a study of passivity of iron and other metals. Preliminary results show with iron a surface potential change of about 0.2 volt in the negative direction upon passivation in concentrated nitric acid followed by a reverse change when made active with hydrochloric acid. This is in the direction that would be expected on the basis of an oxide layer. However, activation by dilute nitric acid gives a more negative value than the passive iron, a result which perhaps means the presence of a different oxide or possibly a basic nitrate.

Acknowledgments are due Doctors L. P. Gotsch and Laubscher for suggesting the use of a *p*H meter for this purpose and Mr. Robert Busch for laboratory assistance. This project was supported in part by the Line Material Company.

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The Measurement of Thermal Radiation at Microwave Frequencies

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The connection between Johnson noise and blackbody radiation is discussed, using a simple thermodynamic model. A microwave radiometer is described together with its theory of operation. The experimentally measured root mean square fluctuation of the output meter of a microwave radiometer (0.4°C) compares favorably with a theoretical value of 0.46°C. With an r-f band width of 16 mc/sec., the 0.4°C corresponds to a minimum detectable power of 10^{-16} watt. The method of calibrating using a variable temperature resistive load is described.

INTRODUCTION

SINCE radio waves may be considered infrared radiation of long wave-length, a hot body would be expected to radiate microwave energy thermally. In order to be a good radiator of microwaves, a body must be a good absorber and the best thermal radiator is the "blackbody."

Although their discoveries were historically unconnected, there is a very close connection between "Johnson noise" of resistors and thermal radiation. The thermal fluctuations of electrons in a resistor set up voltages across the resistor. These "noise voltages" are of such a magnitude that a noise power per unit frequency of kT can be drawn from the resistor. k is Boltzmann's constant; T is the absolute temperature of the resistor.

The connection between thermal radiation and Johnson noise can best be shown by considering the system of Fig. 1.

An antenna is connected to a transmission line which is in turn terminated by a resistor. The radiation impedance of the antenna is assumed to be equal to the characteristic impedance of the coaxial line, i.e., the antenna is "matched" to the line. Also the resistor is assumed to "match" the line. When a transmission line is terminated by a "matched" load, the running waves in the transmission line incident on this load are completely absorbed without reflection. The antenna is completely surrounded by black

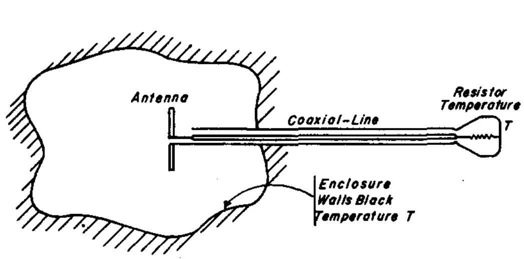

FIG. 1. Antenna system in black enclosure.

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** This paper is based on work done for the Office of Scientific Research and Development under contract OEMsr-262 with the Massachusetts Institute of Technology.

walls of an enclosure. The black enclosure and the resistor are assumed to be at the same temperature T.

Thermal radiation emitted by the walls of the blackbody is picked up by the antenna and transmitted down the line where it is absorbed by the resistor. Johnson noise in the resistor causes a noise power to be transmitted down the line which, passing out the antenna, is absorbed by the black walls. If these two powers were unequal the resistor would either lose or gain energy, resulting in a violation of the second law of thermodynamics. Thus, the available Johnson noise power from a resistor must be equal to the power picked up by an antenna pointed at a blackbody at the same temperature. It is evident that an antenna pointed at surroundings in thermal equilibrium at a temperature T behaves like a resistor at this temperature terminating the coaxial line. It is convenient to describe the radiation intercepted by an antenna in terms o an "antenna temperature." An antenna is said to have a certain temperature when it is receiving an amount of radiation equal to that from a resistor at the temperature in question.

The above arguments were made for a coaxial transmission line, but it is easily seen that they are valid for an antenna fed by any type of single mode transmission line.

The frequency dependence of blackbody radiation differs from that of electrical noise, and this difference may at first glance seem paradoxical. It will be remembered that Planck's blackbody radiation formula reduces to that of Rayleigh-Jeans at the long wave-length end, and in this region the radiation per unit area, per unit solid angle, and per unit frequency of a blackbody is proportional to frequency squared. However, the available noise power from a resistor per unit frequency is frequency independent. The paradox is resolved as follows: in general, the lobes of an antenna pattern become more directive at a higher frequency. It accepts noise from a smaller solid angle; in fact, it can be shown that the average absorption cross section of an antenna is $\lambda^2/4\pi$. The factor λ^2 from the antenna pattern and $1/\lambda^2$ from the blackbody radiation formula just cancel each other, and the total absorbed power is frequency independent.

MEASUREMENT OF THERMAL RADIATION

The detection and measurement of thermal radiation at radiofrequencies requires techniques differing somewhat from those commonly employed for the monitoring of c.w. signals. A c.w. signal, for instance a c.w. carrier, is characterized by a spectrum of small band width, and a receiver for monitoring such a signal is designed with a narrow band. The narrow band rejects noise from frequencies for which there are no signal components, and a good signal to noise ratio is obtained.

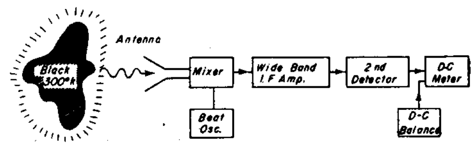

FIG. 2. Wide band receiver used as radiometer.

Thermal radiation, on the other hand, is usually characterized by a spectrum which is very wide. The amplitudes, and phases of different frequency components, are completely independent and are characterized by a complete randomness in the phases. A receiver for the measurement of r-f noise should employ a wide intermediate frequency (i-f) band.

Figure 2 is a block diagram of a wide band receiver used as a radiometer. A microwave receiver somewhat similar to Fig. 2 has been used by Southworth[1] in an interesting series of measurements on microwave radiation from the sun.

It will be assumed for purposes of definiteness that the radiometer of Fig. 2 has an i-f band width of 10^7 cycles/sec., that the noise figure[2] of the receiver is 20, and that the response time of the d.c. output meter is 1 second. The operation of the radiometer is roughly as follows:

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[1] G. C. Southworth, J. Frank. Inst. **239**, 285 (1945).

[2] The noise figure of a receiver is defined by the following operation: A resistor whose absolute temperature is 300°K is connected to the input terminals of the receiver. This resistor is heated until the available noise power from the i-f amplifier is doubled. The noise figure is this change in temperature expressed in units of 300°K. It is implicitly assumed that only a single band of r-f frequencies are converted to the intermediate frequency. If two r-f bands are converted to the intermediate frequency, the noise figure is the change in temperature expressed in units of 150°K. It is evident that the noise figure is a function of the magnitude of the resistance. What is commonly quoted as the noise figure of a receiver is the measured value for the optimum value of resistance.

The receiver will accept signals from two bands separated by twice the intermediate frequency. The thermal radiation picked up by the antenna in these two signal bands is converted to i-f noise in the mixer. Because the receiver

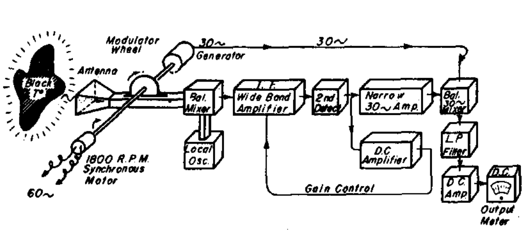

FIG. 3. Microwave radiometer.

is imperfect, noise having its origin in the receiver is added to this converted noise. For the noise figure of 20 assumed above, $\frac{1}{9}$ of the average power entering the second detector has its origin in the 300°K antenna load.

The precision with which thermal radiation can be measured is limited by the statistical fluctuation in the output meter. The order of magnitude of these fluctuations can be obtained from the following argument. With an i-f bandwidth of 10^7 cycles/sec., about 10^7 rectified noise pulses leave the second detector each second. Because of the statistical nature of noise, some of these pulses are large and some are small, but the probability of a given pulse being large or small is completely independent of the sizes of preceding pulses. This situation can be simplified somewhat without changing the basic ideas. It will be assumed that these noise pulses appear like beads on a string and are of two sizes, large and small, in essentially equal numbers. It will be assumed that the small pulses are so small that they do not contribute to the rectified current registered by the d.c. meter assumed to have a time constant of one second. The d.c. meter reading is a number which is essentially the number of big pulses in the preceding second. The problem is now equivalent to flipping pennies a large number of times, always counting the number of heads which appear. It will be remembered that in $2n$ tosses of a coin, the number of heads which appear is likely to differ from n by a number of the order of $(n)^{\frac{1}{2}}$. The implication is fairly obvious. The fluctuation in the number of pulses counted by the d.c. meter meter in each second is about 10^7, and the fluctuation in the d.c. meter reading is about $1/(10^7)^{\frac{1}{2}} = 3\cdot 10^{-4}$ of the meter reading. Since for the noise figure assumed above the radiation from a 300°K load produces about $\frac{1}{10}$ of the d.c. output, the fluctuation in the output meter is about

$$300° \times 10 \times 3\cdot 10^{-4} \sim 1°\text{C}.$$

Thus it should be possible to measure the temperature of an antenna termination to an accuracy of about 1°C. It should be noted that this fluctuation temperature varies inversely as the square root of the band width, and hence, the wider the i-f band the better the radiometer. The fluctuation temperature also varies directly as the square root of the band width of the d.c. output meter. The more sluggish the meter, the smaller the fluctuations but also, unfortunately, the more time required to make a measurement.

The above discussion was, of course, oversimplified but an exact calculation gives numbers of the same order of magnitude. There is, however, an important source of fluctuation which was intentionally omitted. If the gain of the receiver varies in a random way, the output meter will fluctuate, and this type of fluctuation is likely to be larger than the other. The more obvious sources of these fluctuations are, of course, line voltage variations, and ambient temperature drifts. However, even if power supplies are carefully regulated and temperatures stabilized there are fluctuations having their origins in the tubes themselves. Also fluctuations in the conversion gain and intrinsic noise of a

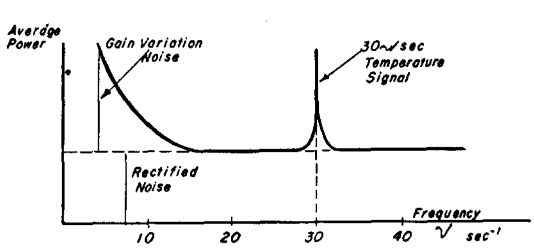

FIG. 4. Second detector output.

crystal mixer must be included. Since a variation of the over-all gain of only 0.1 percent produces a change of 3°C on the output meter, it can be seen that this type of fluctuation is serious. Even though some of these sources of added fluctuation do not have their origin in gain variations, it will be helpful to call all low frequency fluctuations over and above the regular rectified noise "gain variation noise." The regular calculable low frequency noise from the second detector will be called "rectified noise."

THE MICROWAVE RADIOMETER

In order to eliminate "gain variation noise," the radiometer shown in block diagram in Fig. 3 was designed.

The device shown in Fig. 3 consists of an antenna (shown as a horn) connected to a crystal balanced mixer (see Fig. 5 for details of mixer) by a rectangular wave guide. The connecting wave guide contains a slotted section in which an absorbing wheel is caused to rotate. The absorbing wheel is driven by a motor at 30 cycles/sec. and is so shaped that it produces a nearly square modulation with about equal times in and out of the guide. It is assumed that both the wheel and antenna are non-reflecting. Under these conditions the effect of inserting the wheel is one of disconnecting the antenna and connecting an equivalent resistance (represented by the absorbing wheel). If the amount of radiation (or noise power) produced by the wheel while it is in the guide is the same as that being picked up by the antenna, this commutation will produce no change in the output of the second detector. However, if the radiation from the wheel is either more or less than that from the antenna, the output of the second detector will modulate up and down at 30 cycles/sec.

The 30-cycle component of this modulation is amplified by the 30-cycle amplifier and then mixed with 30 cycles from the coherent beat frequency generator to produce d.c. The output from the 30-cycle balanced mixer is fed through a very narrow band low pass filter into a d.c. amplifier which drives a d.c. meter. This d.c. meter measures the difference between the antenna temperature and the modulating wheel's temperature. The meter needle swings either to the left or the right depending upon the sign of this difference.

The device of Fig. 3 has the advantage over that of Fig. 2 of eliminating the "gain variation noise" from the output meter. The reason for the elimination of this type of fluctuation can be understood, at least qualitatively from the following arguments: in the radiometer of Fig. 3 the radiation from the antenna is compared with that generated by a standard reference resistor (the modulator wheel). This comparison is made

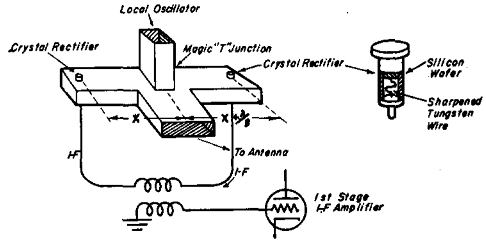

FIG. 5. Microwave balanced mixer with wave guide circuits.

30 times a second and in the short time interval of 1/30 sec. the amplifier gain does not change greatly. The output meter with its long time constant is effectively measuring the average of a great number of individual measurements, each requiring 1/30 of a second to perform.

In Fig. 4 there is plotted the average power against frequency for the output of the second detector of Fig. 3. It is to be noted that the signal, appearing as it does at 30 cycles/sec., avoids the "gain variation noise."

It has been assumed in the description of the operation of Fig. 3 that the antenna and modulating wheel presented equal impedances to the receiver. This equality can, of course, never be perfectly realized and it is necessary to examine the effects introduced by small mismatches in these components. For the purpose of this discussion, it will be assumed that the wheel is a perfect match but that the antenna is slightly mismatched.

There are several possible sources of error due to antenna mismatch but the most important of these seems to be the i-f impedance effect. The i-f impedance of a single crystal mixer depends to a marked degree upon the r-f circuit impedances presented to the mixer. Radiofrequency impedances are transformed through the mixer into i-f impedances, and a 30-cycle/sec. variation in the r-f impedances is transformed into a 30-cycle/sec. variation in the i-f impedance presented to the i-f amplifier. As the gain and noise figure of the i-f amplifier depend upon the i-f impedance of the input load, there is generated a variation in the output of the second detector as a result of this effect.

By employing two crystals in the proper r-f and i-f circuits, a mixer can be constructed which does not convert from i-f to r-f and hence for which changes in the r-f impedances are not reflected into the i-f circuit. Figure 5 is a diagram of such a microwave balanced mixer employing wave guide r-f circuits. In general, it is desirable to make one of the wave guides feeding a crystal rectifier of adjustable length and to include tuners for the two crystal rectifiers.

In addition to minimizing effects arising from impedance transformation to the i-f circuit, the balanced mixer eliminates converted local oscillator noise and minimizes the local oscillator power that enters the antenna. It is desirable to keep local oscillator power out of the antenna line as the load is strictly speaking not in thermal equilibrium with an external source of power being dissipated in it. Effects have been observed which had their origin in poor contacts modulating the local oscillator power to produce noise side bands.

As was pointed out previously, the larger the band width of the i-f amplifier the smaller the fluctuation in the output meter. However, if this band width is too great the noise figure of the i-f amplifier suffers, and there is a certain optimum band width for crystals of particular noise temperature. With present techniques in the microwave region, this optimum is probably in the neighborhood of 10 mc. There is presumably also an optimum band shape for the i-f amplifier, but this question is obscured by a number of unknown factors. If the noise figure of the amplifier is constant over the pass band of the amplifier, a square band pass is the best.

The band width and shape of the low pass filter circuit of Fig. 3 is important in determining the performances of the system. As was mentioned before, the narrower this pass band, the smaller the fluctuation in the output meter. However, this narrowing can only be accomplished at the expense of a sluggish meter with resulting long observation time.

The optimum band shape for the low pass filter is that which gives the most rapid meter response for a given band width. Instead of con-

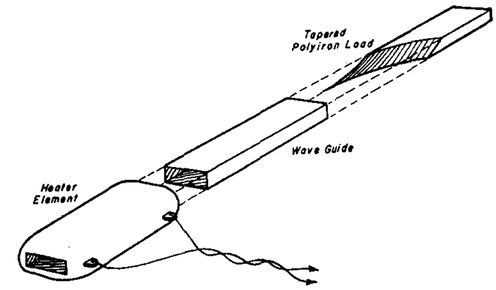

FIG. 6. Exploded view of calibrating load (thermometer not shown).

sidering the most general problem the following simplified calculation has been made: How must a galvanometer be damped to make its response to a step function input as rapid as possible keeping the noise fluctuation constant? The result, as might be expected, is that the critically damped galvanometer is best.

The low pass filter of Fig. 3 is so designed that the d.c. meter has all the characteristics of a long time constant, critically damped galvanometer—all this without broken suspensions.

In Appendix 1, the root mean square temperature fluctuation of the output meter is calculated assuming that all "gain variation noise" is absent.[3] The result for a typical set of system parameters is

$$[\langle(\Delta T)^2\rangle_{Av}]^{\frac{1}{2}} = 0.46°\text{C}.$$

The system parameters corresponding to this fluctuation are:

Modulator wheel—square wave
Receiver noise figure—25
I-f band width—8 mc/sec.
I-f band shape—square
Second detector—linear
Time constant of low pass filter—2.5 sec.
Band shape of low pass filter—critically damped.

The actual root mean square temperature fluctuations measured for a system having approximately the above parameters was 0.4°.

The power sensitivity of the radiometer with the above parameters is of some interest. The device will detect $\frac{1}{2}$°C and with the total r-f band width of 16 mc/sec. the device has a minimum detectable power of

$$P = kT\Delta\nu = 10^{-16} \text{ watt}.$$

[3] The writer is greatly indebted to Dr. G. E. Uhlenbeck and Miss M. Wang for valuable aid in connection with this problem.

RADIOMETER CALIBRATION

A microwave radiometer can be calibrated by using an r-f absorbing load, the temperature of which can be varied. The absorbing load is substituted for the horn antenna in the calibrating measurement. The calibrating load used in a series of experiments consisted of a section of wave guide containing a tapered "polyiron"[4] absorbing load. A heater element, consisting of resistance wire wrapped around the guide, was used to heat the load. A bi-metallic thermometer was used to measure the temperature of the wave guide. (See Fig. 6 for an exploded view of the calibrating load.) In practice, the temperature of the guide was raised about 60°C and the deflection in the output meter noted.

It is evident that there are many uses to which a microwave radiometer may be put. As an example, an r-f noise source may be calibrated referring to thermal noise as a standard. A noise source so calibrated may be used to make absolute noise figure measurements.

A radiometer operating in a region slightly less than 1 cm^{-1} has been used by R. Beringer and the author to make some observations on radiation from the sun and moon. These results will be reported elsewhere.

Microwave radiometers have also been used to measure atmospheric absorption at several radio frequencies. An account of these experiments will be given in another paper by R. Beringer, R. L. Kyle, A. B. Vane, and the author.

ACKNOWLEDGMENTS

The size of the Radiation Laboratory being what it was, it is impossible to acknowledge all the people who have aided in this development. However, special acknowledgment is due the authors colleagues, Dr. R. Beringer, Mr. R. L. Kyle, and Mr. A. B. Vane, for their active participation in the development of the radiometer. Special thanks are also due Mr. C. F. Nawrocki for developing a special oscillator to generate the beat frequency for one of the radiometers. Finally, the author wishes to express his appreciation to Dr. E. M. Purcell for his continued interest, support, and advice.

[4] "Polyiron" is finely divided iron held together with a plastic binder.

APPENDIX 1

Statistical Fluctuations of Output Meter

Assume a unit of time, τ seconds, such that $2\pi\tau$ seconds is much greater than any correlation time of apparatus. All noise voltages will be expanded in a Fourier series of this period.

Let the frequency response function of the i-f amplifier be

$$F(\omega), \tag{1}$$

where $\omega = 2\pi \times$frequency in cycles/τ sec.

An alternative representation for $F(\omega)$ in terms of a Fourier series is

$$F_n = F(n). \tag{2}$$

n is an integer.

Let the response function of the low pass filter be

$$S(\omega) \text{ or } S_n = S(n). \tag{3}$$

Case I

Assume that the modulating wheel and the source are at the same temperature so that there is no temperature signal present.

Let $F(t)$ be the output of the i-f amplifier for some $2\pi\tau$ second period where t is again in units of $2\pi\tau$ seconds.

$$F(t) = \sum_n f_n e^{int}. \tag{4}$$

Assume for the time being that the second detector is square law. The output from the second detector is

$$g(t) = F^2(t) = \sum_n G_n e^{int}, \quad = \sum_{n,m} f_n f_m e^{i(n+m)t}. \tag{5}$$

Imagine the above function obtained for a large number of periods each of $2\pi\tau$ seconds. An ensemble average will be an average of corresponding quantities from each of these periods.

If the input r-f noise spectrum is essentially flat over the i-f pass band, the ensemble averages:

$$\langle f_n f_{-n}\rangle_{\text{Av}} = PF_nF_{-n} = P|F_n|^2, \tag{6}$$
$$\langle f_n f_{-m}\rangle_{\text{Av}} = 0 \quad n \neq m,$$

where P is a constant and

$$\langle G\rangle_{\text{Av}} = P\sum_n |F_n|^2; \tag{7}$$

also,

$$\langle |G_n|^2\rangle_{Av} = 2P^2 \sum_m |F_m|^2 |F_{m+n}|^2. \quad (8)$$

Let the output from the second mixer after passing through the low pass filter be

$$S(t) = \sum_n S_n(G_{n+k} + G_{n-k})e^{int}, \quad (9)$$

where k is the modulation frequency. Form $S^2(t)$ and take the time average over the period

$$\langle S^2(t)\rangle_{Av} = \sum_n |S_n|^2 \cdot |G_{n+k} + G_{n-k}|^2. \quad (10)$$

Form the ensemble average of (10)

$$\begin{aligned}\langle\langle S^2(t)\rangle_{Av}\rangle_{Av} = \langle S^2(t)\rangle_{Av} &= \sum_n |S_n|^2 \cdot \langle |G_{n+k}|^2\rangle_{Av} + \langle |G_{n-k}|^2\rangle_{Av}, \\ &= 2P^2 \sum_n |S_n|^2 \cdot [\sum_m |F_m|^2 \cdot |F_{m+n+k}|^2 + \sum_m |F_m|^2 \cdot |F_{m+n-k}|^2]. \quad (11)\end{aligned}$$

For small n

$$|F_m| \cong |F_{m+n}|,$$

$$\langle S^2(t)\rangle_{Av} = 4P^2 \sum_{n,m} |S_n|^2 \cdot |F_m|^4. \quad (12)$$

Case II

Let the antenna load temperature differ from the wheel temperature T by ΔT. Assume a noise figure of N for the receiver. The noise power into the second detector varies with the time as the modulating wheel rotates. It will be assumed that this modulation is square wave and that the power into the second detector is

$$P = P_0 + p(t), \quad (13)$$

where $p(t)$ is a square wave function of the time t as shown in Fig. 7 and

$$p_0 = \frac{\Delta T}{T}\frac{P_0}{N}. \quad (14)$$

Equation (13) may be expanded in a Fourier series and becomes

$$P = \sum_n p_n e^{int},$$

$$p_n = 2p_0/\pi|n|. \quad (15)$$

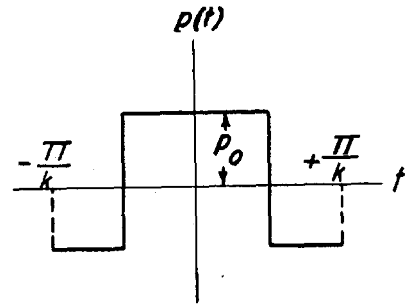

FIG. 7. $p(t)$ square wave function of t.

With the temperature signal present, $n \neq 0$ Eq. (9) becomes modified as follows:

$$S(t) = \sum_n S_n(G_{n+k} + G_{n-k})e^{int} + 2S_0 \cdot G_0 \cdot \frac{P_1}{P_0}. \quad (16)$$

The last term on the right of (16) is the contribution caused by signal and its square is

$$(\text{Signal})^2 = 4S_0^2 G_0^2 \frac{P_1^2}{P_0^2}. \quad (17)$$

Substituting (7) in (17)

$$(\text{Signal})^2 = 4S_0^2 p_1^2 (\sum_n |F_n|^2)^2, \quad (18)$$

since $P = P_0$ for no temperature signal. Equation (12) is the noise fluctuation and (18) divided by (12) becomes

$$\frac{(\text{Signal})^2}{(\text{Noise})^2} = \frac{P_1^2}{P_0^2} \frac{S_0^2(\sum_n |F_n|^2)^2}{(\sum |S_n|^2)\cdot(\sum_m |F_m|^4)}. \quad (19)$$

Substituting from (14) and (15) and writing the sums in the form of integrals

$$\frac{(\text{Signal})^2}{(\text{Noise})^2} = \frac{4}{\pi^2}\left(\frac{\Delta T}{T}\right)^2 \frac{1}{N^2} \times \frac{[\int |F(\omega)|^2 d\omega]^2 \cdot S^2(0)}{[\int |F(\omega)|^4 d\omega]\cdot[\int |S(\omega)|^2 d\omega]}. \quad (20)$$

For a signal/noise = 1, (20) becomes

$$\frac{\Delta T}{T} = \frac{\pi N}{2} \frac{\left\{\left[\int_{-\infty}^{\infty} |F(\omega)|^4 d\omega\right]\cdot\left[\int_{-\infty}^{\infty} |S(\omega)|^2 d\omega\right]\right\}^{\frac{1}{2}}}{S(0)\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega}. \quad (21)$$

To state (21) in words, ΔT is the root mean square temperature fluctuation of the output meter in terms of the system parameters.

Dr. O. E. Uhlenbeck[5] has shown that for an ideal linear detector, the output fluctuation is just $\frac{1}{2}$ that of the square law detector. For a linear detector (21) becomes

$$\frac{\Delta T}{T}=\frac{\pi N}{4}\frac{\left\{\left[\int_{-\infty}^{\infty}|F(\omega)|^4d\omega\right]\cdot\left[\int_{-\infty}^{\infty}|S(\omega)|^2d\omega\right]\right\}^{\frac{1}{2}}}{S(0)\int_{-\infty}^{\infty}|F(\omega)|^2d\omega}. \tag{22}$$

Equations (21) and (22) are dimensionless and are valid for any unit of time measure.

Example: Assume

Receiver noise figure—$N=25$
I-f band shape—square
I-f band width—$\Delta\omega=5.10^7$ sec.$^{-1}$
Second detector—linear
Band shape of low pass filter—critically damped
Band width of low pass filter—.4 sec.$^{-1}$

The response function of a critically damped filter is of the form

$$S(\omega)=-\frac{a^2}{(\omega-ia)^2}, \tag{23}$$

where in this case $a=.4$ sec.$^{-1}$, the integral

$$\int_{-\infty}^{\infty}|S(\omega)|^2d\omega=\frac{\pi a}{2}, \tag{24}$$

and

$$S(0)=1. \tag{25}$$

Also for a square i-f band of width $\Delta\omega$

$$\frac{\left(\int_{-\infty}^{\infty}|F(\omega)|^4d\omega\right)^{\frac{1}{2}}}{\int_{-\infty}^{\infty}|F(\omega)|^2d\omega}=\frac{1}{(2\Delta\omega)^{\frac{1}{2}}}. \tag{26}$$

Substituting (24)–(26) in (22)

$$\frac{\Delta T}{T}=\frac{\pi^{\frac{3}{2}}N}{8}\left(\frac{a}{\Delta\omega}\right)^{\frac{1}{2}}. \tag{27}$$

Substituting numerical values, assuming that $T=300°$K

$$\Delta T=0.46°\text{C}. \tag{28}$$

[5] Private communication.