Chapter 3
Place, time, and motion

> Then, just for a minute... he turned off the lights.... And then while we all still waited I understood that the terror of my dream was not about losing just vision, but the whole of myself, whatever that was. What you lose in blindness is the space around you, the place where you are, and without that you might not exist. You could be nowhere at all.
>
> – Barbara Kingsolver, *Animal Dreams*, 1990

Where is Mars? The center of our Galaxy? The brightest X-ray source? Where, indeed, are we? Astronomers have always needed to locate objects and events in space. As our science evolves, it demands ever more exact locations. Suppose, for example, an astronomer observes with an X-ray telescope and discovers a source that flashes on and off with a curious rhythm. Is this source a planet, a star, or the core of a galaxy? It is possible that the X-ray source will appear to be quite unremarkable at other wavelengths. The exact position for the X-ray source might be the only way to identify its optical or radio counterpart. Astronomers need to know where things are.

Likewise, knowing *when* something happens is often as important as *where* it happens. The rhythms of the spinning and orbiting Earth gave astronomy an early and intimate connection to timekeeping. Because our Universe is always changing, astronomers need to know what time it is.

The "fixed stars" are an old metaphor for the unchanging and eternal, but positions of real celestial objects do change, and the changes tell stories. Planets, stars, gas clouds, and galaxies all trace paths decreed for them. Astronomers who measure these motions, sometimes only through the accumulated labors of many generations, can find in their measurements the outlines of nature's decree. In the most satisfying cases, the measurements uncover fundamental facts, like the distances between stars or galaxies, or the age of the Universe, or the presence of planets orbiting other suns beyond the Sun. Astronomers need to know how things move.

3.1 Astronomical coordinate systems

> Any problem of geometry can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for its construction.
>
> – Rene Descartes, *La Geometrie, Book I*, 1637

Descartes' brilliant application of coordinate systems to solve geometric problems has direct relevance to astrometry, the business of locating astronomical objects. Although astrometry has venerably ancient origins,[1] it retains a central importance in astronomy.

3.1.1 Three-dimensional coordinates

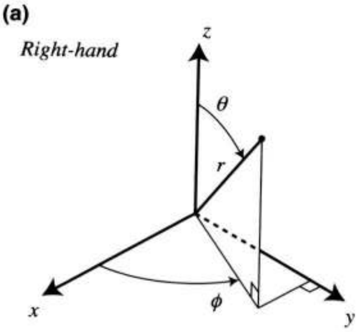
I assume you are familiar with the standard (x, y, z) Cartesian coordinate system and the related spherical coordinate system (r, ϕ, θ), illustrated in Figure 3.1(a). Think for a moment how you might set up such a coordinate system in practice. Many methods could lead to the same result, but consider a process that consists of four decisions:

1. Locate the origin. In astronomy, this often corresponds to identifying some distinctive real or idealized object: the centers of the Earth, Sun, or Galaxy, for example.
2. Locate the x–y plane. We will call this the "fundamental plane." The fundamental plane, again, often has physical significance: the plane defined by the Earth's equator – or the one that contains Earth's orbit – or the symmetry plane of the Galaxy, for example. The z-axis passes through the origin perpendicular to the fundamental plane.
3. Decide on the direction of the positive x-axis. We will call this the "reference direction." Sometimes the reference direction has a physical significance – the direction from the Sun to the center of the Galaxy, for example. The y-axis then lies in the fundamental plane, perpendicular to the x-axis.
4. Finally, decide on a convention for the signs of the y- and z-axes. These choices produce either a left- or right-handed system – see below."

The traditional choice for measuring the *angles* is to measure the first coordinate, ϕ (or λ), within the fundamental plane so that ϕ increases from the $+x$-axis towards the $+y$-axis (see Figure 3.1). The second angle, θ (or ζ), is measured in a plane perpendicular to the fundamental plane increasing from the positive z axis towards the x–y plane. In this scheme, ϕ ranges, in radians, from 0 to 2π and θ ranges from 0 to π. A common alternative is to measure the second angle (β in the figure) from the x–y plane, so it ranges between $-\pi/2$ and $+\pi/2$.

[1] Systematic Babylonian records go back to about 650 BC but with strong hints that the written tradition had Sumerian roots in the late third millennium. Ruins of megalithic structures with clear astronomical alignments date from as early as 4500 BC (Nabta, Egypt).

(a)



(b)

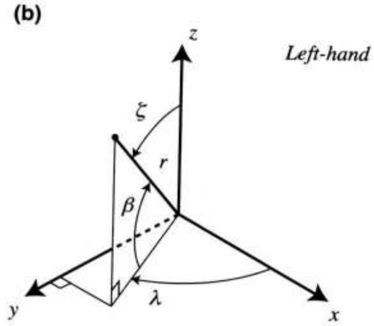


Fig. 3.1 Three-dimensional coordinate systems. (a) The traditional system is right-handed. (b) This system is left-handed, its axes are a mirror image of those in (a). In either system one can choose to measure the second angle from the fundamental plane (e.g. angle β) instead of from the z axis (angles θ or ζ).

The freedom to choose the signs of the y- and z-axes in step 4 of this procedure implies that there are two (and only two) kinds of coordinate systems. One, illustrated in Figure 3.1a, is ***right-handed***: if you wrap the fingers of your right hand around the z axis so the tips point in the $+\phi$ direction (that is, from the $+x$ axis towards the $+y$ axis), then your thumb will point in the $+z$ direction. In a ***left-handed*** system, like the (r, λ, ζ) system illustrated in Figure 3.1(b), you use your left hand to find the $+z$ direction. The left-handed system is the mirror image of the right-handed system. In either system, Pythagoras gives the radial coordinate as:

$$r = \sqrt{x^2 + y^2 + z^2}$$

3.1.2 Coordinates on a spherical surface

> It is one of the things proper to geography to assume that the Earth as a whole is spherical in shape, as the universe also is...
>
> – Strabo, *Geography*, II, 2, 1, *c*. AD 18

If all points of interest are on the surface of a sphere, the r coordinate is superfluous, and we can specify locations with just two angular coordinates like (ϕ, θ) or (λ, β). Many astronomical coordinate systems fit into this category, so it is useful to review some of the characteristics of geometry and trigonometry on a spherical surface.

1. A ***great circle*** is formed by the intersection of the sphere and a plane that contains the center of the sphere. The shortest distance between two points on the surface of a sphere is an arc of the great circle connecting the points.
2. A ***small circle*** is formed by the intersection of the sphere and a plane that does not contain the center of the sphere.
3. The ***spherical angle*** between two great circles is the angle between the planes, or the angle between the straight lines tangent to the two great circle arcs at either of their points of intersection.
4. A ***spherical triangle*** on the surface of a sphere is one whose sides are all segments of great circles. Since the sides of a spherical triangle are arcs, the sides can be measured in angular measure (i.e. radians or degrees) rather than linear measure. See Figure 3.2.
5. The ***law of cosines*** for spherical triangles in Figure 3.2 is:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

or

$$\cos A = \cos B \cos C + \sin B \sin C \cos a$$

6. The ***law of sines*** is

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

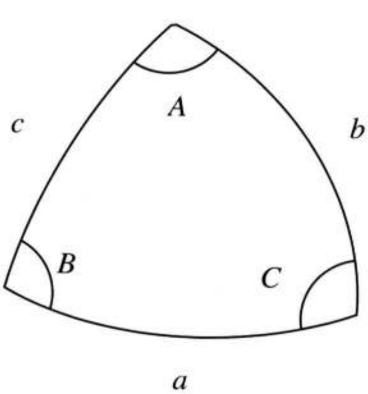


Fig. 3.2 A spherical triangle. You must imagine this figure is drawn on the surface of a sphere. A, B, and C are spherical angles; a, b, and c are arcs of great circles.

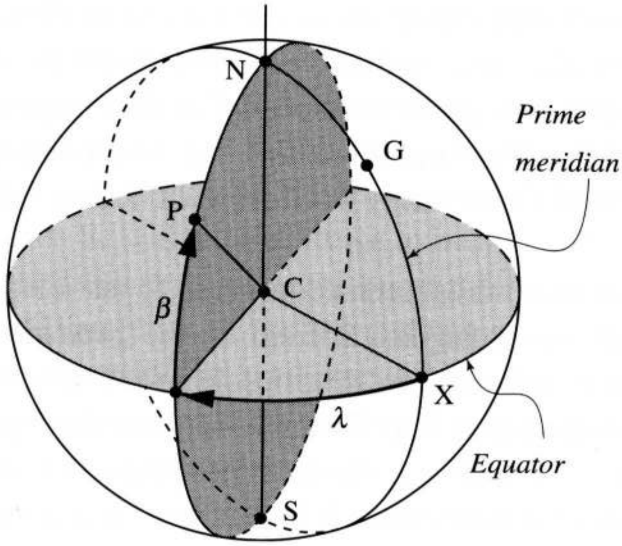


Fig. 3.3 The latitude–longitude system. The center of coordinates is at C. The fundamental direction, line CX, is defined by the intersection of the prime meridian (great circle NGX) and the equator. Latitude, β, and longitude, λ, for some point, P, are measured as shown. Latitude is positive north of the equator, negative south. Astronomical longitude for Solar System bodies is positive in the direction opposite the planet's spin. (i.e. to the west on Earth). On Earth, coordinates traditionally carry no algebraic sign, but are designated as north or south latitude, and west or east longitude. The coordinate, β, is the geocentric latitude. The coordinate actually used in practical systems is the geodetic latitude (see the text).

3.1.3 Terrestrial latitude and longitude

> "I must be getting somewhere near the center of the Earth. . .yes. . .but then I wonder what Latitude and Longitude I've got to?" (Alice had not the slightest idea what Latitude was, nor Longitude either, but she thought they were nice grand words to say.)
>
> – Lewis Carroll, *Alice's Adventures in Wonderland*, 1897

Ancient geographers introduced the seine-like latitude–longitude system for specifying locations on Earth well before the time Hipparchus of Rhodes (*c*. 190–120 BC) wrote on geography. Figure 3.3 illustrates the basic features of the system.

In our scheme, the first steps in setting up a coordinate system are to choose an origin and fundamental plane. We can understand why Hipparchus, who believed in a geocentric cosmology, would choose the center of the Earth as the origin. Likewise, choice of the equatorial plane of the Earth as the fundamental plane makes a lot of practical sense. Although the location of the equator may not be obvious to a casual observer like Alice, it is easily determined from simple astronomical observations. Indeed, in his three-volume book on geography, Eratosthenes of Alexandria (*c*. 275 – *c*. 194 BC) is said to have computed the location of the equator relative to the parts of the world known to him. At the time, there was considerable dispute as to the habitability of the (possibly too hot) regions near the equator, but Eratosthenes clearly had little doubt about their location.

Great circles perpendicular to the equator must pass through both poles, and such circles are termed ***meridians***. The place where one of these – the ***prime meridian*** – intersects the equator could constitute a reference direction (x-axis). Unfortunately, on Earth, there is no obvious meridian to use for this purpose. Many choices are justifiable, and for a long time geographers simply chose a prime meridian that passed though some locally prominent or worthy place. Thus, the latitude of any point on Earth was unique, but its longitude was not,

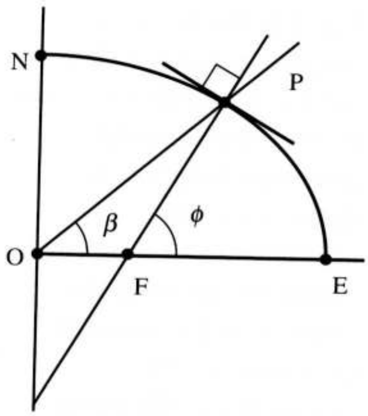


Fig. 3.4. Geocentric (β) and geodetic (ϕ) latitudes. Line PF is perpendicular to the surface of the reference spheroid, and approximately in the direction of the local vertical (local gravitational force).

since it depended on which meridian one chose as prime. This was inconvenient. Eventually, in 1884, the "international" community (in the form of representatives of 25 industrialized countries meeting in Washington, DC, at the First International Meridian Conference) settled the zero point of longitude at the meridian of the Royal Observatory in Greenwich, located just outside London, England.

You should note that the latitude coordinate, β, just discussed, is called the ***geocentric latitude***, to distinguish it from ϕ, the ***geodetic latitude***. Geodetic latitude is defined in reference to an ellipsoid-of-revolution that approximates the actual shape of the Earth. It is the angle between the equatorial plane and a line perpendicular to the surface of the reference ellipsoid at the point in question.

Figure 3.4 shows the north pole, N, equator, E, and center, O, of the Earth. The geocentric and geodetic latitudes of point P are β and ϕ, respectively. Geodetic latitude is easier to determine and is the one employed in specifying positions on the Earth. The widely used technique of global positioning satellites (GPS), for example, returns geodetic latitude, longitude, and height above a reference ellipsoid. To complicate things a bit more, the most easily determined latitude is the ***geographic latitude***, the angle between the local vertical and the equator. Massive objects like mountains affect the geographic but not the geodetic latitude and the two can differ by as much as an arc minute. Further complications on the sub-arc-second scale arise from short- and long-term motion of the geodetic pole itself relative to the Earth's crust due to tides, earthquakes, internal motions, and continental drift.

Planetary scientists establish latitude–longitude systems on other planets, with latitude usually easily defined by the object's rotation, while definition of longitude depends on identifying some feature to mark a prime meridian.

Which of the two poles of a spinning object is the "north" pole? In the Solar System, the preferred (but not universal!) convention is that the ***ecliptic*** – the plane containing the Earth's orbit – defines a fundamental plane, and a planet's north pole is the one that lies to the (terrestrial) north side of this plane. Longitude should be measured as increasing in the direction opposite the spin direction.

For other objects, practices vary. One system says the north pole is determined by a right-hand rule applied to the direction of spin: wrap the fingers of your right hand around the object's equator so that they point in the direction of its spin. Your thumb then points north (in this case, "north" is in the same direction as the angular momentum vector).

3.1.4 The altitude–azimuth system

Imagine an observer, a shepherd with a well-behaved flock, say, who has some leisure time on the job. Our shepherd is lying in an open field, contemplating the sky. After a little consideration, our observer comes to imagine the sky as a hemisphere – an inverted bowl whose edges rest on the horizon. Astronomical objects, whatever their real distances, can be seen to be stuck onto or projected onto the inside of this hemispherical sky.

This is another situation in which the *r*-coordinate becomes superfluous. The shepherd will find it difficult or impossible to determine the *r* coordinate for the objects in the sky. He knows the direction of a star but not its distance from the origin (which he will naturally take to be himself). Astronomers often find themselves in the same situation as the shepherd. A constant theme throughout astronomy is the problem of the third dimension, the *r*-coordinate: the directions of objects are easily and accurately determined, but their distances are not. This prompts us to use coordinate systems that ignore the *r*-coordinate and only specify the two direction angles.

In Figure 3.5, we carry the shepherd's fiction of a hemispherical sky a little bit further, and imagine that the hemispherical bowl of the visible sky is matched by a similar hemisphere below the horizon, so that we are able to apply a spherical coordinate scheme like the one illustrated. Here, the origin of the system is at O, the location of the observer. The fundamental plane is that of the "flat" Earth (or, to be precise, a plane tangent to the tiny spherical Earth at point O). This fundamental plane intersects the sphere of the sky at the ***celestial horizon*** – the great circle passing through the points NES in the figure. ***Vertical circles*** are great circles on the spherical sky that are perpendicular to the fundamental plane. All vertical circles pass through the overhead point, which is called the ***zenith*** (point T in the figure), as well as the diametrically opposed point, called the ***nadir***. The vertical circle that runs in the north–south direction (circle NTS in the figure) is called the ***observer's meridian***.

The fundamental direction in the altitude–azimuth coordinate system runs directly north from the observer to the intersection of the meridian and the celestial horizon (point N in the figure). In this system, a point on the sky, P, has two coordinates:

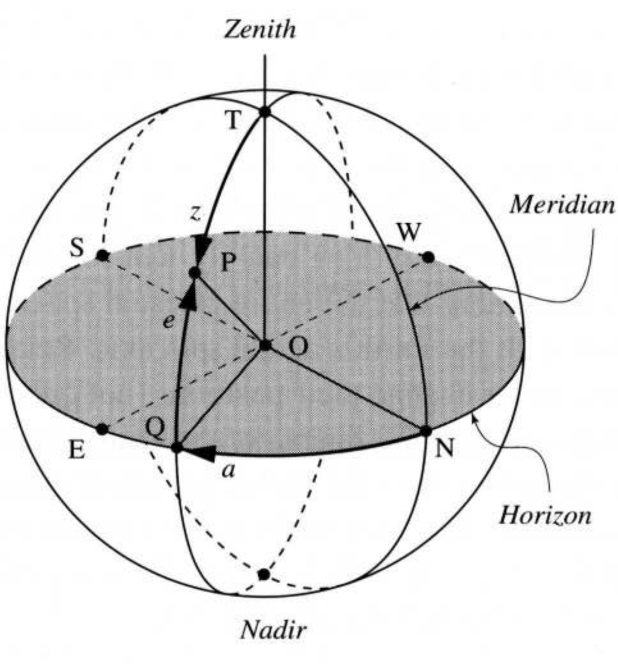


Fig. 3.5. The altitude–azimuth system. The horizon defines the fundamental plane (gray) and the north point on the horizon, N, defines the fundamental direction. Point P has coordinates *a* (azimuth), which is measured along the horizon circle from north to east, and *e* (altitude), measured upwards from the horizon. Objects with negative altitudes are below the horizon.

- The **altitude**, or **elevation**, is the angular distance of P above the horizon (∠QOP or e in the figure). Objects below the horizon have negative altitudes.
- The **azimuth** is the angular distance from the reference direction (the north point on the horizon) to the intersection of the horizon and the vertical circle passing through the object (∠NOQ or a in the figure).

Instead of the altitude, astronomers sometimes use its complement, z, the ***zenith distance*** (∠TOP in the figure).

The (a, e) coordinates of an object clearly describe where it is located in an observer's sky. You can readily imagine an instrument that would measure these coordinates: a telescope or other sighting device mounted to rotate on vertical and horizontal circles that are marked with precise graduations.

One of the most elementary astronomical observations, noticed even by the most unobservant shepherd, is that celestial objects don't stay in the same place in the horizon coordinate system. Stars, planets, the Sun, and Moon all execute a ***diurnal motion***: they rise in the east, cross the observer's meridian, and set in the west. This, of course, is a reflection of the spin of our planet on its axis. The altitude and azimuth of celestial objects will change as the Earth executes its daily rotation. Careful measurement will show that stars (but not the Sun and planets, which move relative to the "fixed" stars) will take about 23 hours, 56 minutes and 4.1 seconds between successive meridian crossings. This period of time is known as one ***sidereal day***. Very careful observations would show that the sidereal day is actually getting longer, relative to a stable atomic clock, by about 0.0015 second per century. The spin rate of the Earth is slowing down.

3.1.5 The equatorial system: definition of coordinates

Because the altitude and azimuth of celestial objects change rapidly, we create another reference system, one in which the coordinates of stars remain the same. In this ***equatorial coordinate system***, we carry the fiction of the spherical sky one step further. Imagine that all celestial objects were stuck on a sphere of very large radius, whose center is at the center of the Earth. Furthermore, imagine that the Earth is insignificantly small compared to this ***celestial sphere***. Now adopt a geocentric point of view. You can account for the diurnal motion of celestial objects by presuming that the entire celestial sphere spins east to west on an axis coincident with the Earth's actual spin axis. Relative to one another objects on the sphere never change their positions (not quite true – see below). The star patterns that make up the figures of the constellations stay put, while terrestrials observe the entire sky – the global pattern of constellations – to spin around its north–south axis once each sidereal day. Objects stuck on the celestial sphere thus appear to move east to west across the terrestrial sky, traveling in small circles centered on the nearest celestial pole.

The fictional celestial sphere is an example of a scientific model. Although the model is not the same as the reality, it has features that help one discuss, predict, and understand real behavior. (You might want to think about the meaning of the word "understand" in a situation where model and reality differ so extensively.) The celestial-sphere model allows us to specify the positions of the stars in a coordinate system, the equatorial system, which is independent of time, at least on short scales. Because positions in the equatorial coordinate system are also easy to measure from Earth, it is the system astronomers use most widely to locate objects on the sky.

The equatorial system *nominally* chooses the center of the Earth as the origin and the equatorial plane of the Earth as the fundamental plane. This aligns the z-axis with the Earth's spin axis, and fixes the locations of the two ***celestial poles*** – the intersection of the z-axis and the celestial sphere. The great circle defined by the intersection of the fundamental plane and the celestial sphere is called the ***celestial equator***. One can immediately measure a latitude-like coordinate with respect to the celestial equator. This coordinate is called the ***declination*** (abbreviated as Dec or δ), whose value is taken to be zero at the equator, and positive in the northern celestial hemisphere; see Figure 3.6.

We choose the fundamental direction in the equatorial system by observing the motion of the Sun relative to the background of "fixed" stars. Because of the Earth's orbital motion, the Sun appears to trace out a great circle on the celestial sphere in the course of a year. This circle is called the ***ecliptic*** (it is where eclipses happen) and intersects the celestial equator at an angle, ϵ, called the ***obliquity of the ecliptic***, equal to about 23.5 degrees. The point where the Sun crosses the equator traveling from south to north is called the ***vernal equinox*** and this point specifies the reference direction of the equatorial system. The coordinate angle measured in the equatorial plane is called the ***right ascension*** (abbreviated as RA or α). As shown in Figure 3.6, the equatorial system is right-handed, with RA increasing from west to east.

For reasons that will be apparent shortly, RA is usually measured in hours: minutes:seconds, rather than in degrees (24 hours of RA constitute 360 degrees of arc at the equator, so one hour of RA is 15 degrees of arc long at the equator). To deal with the confusion that arises from both the units of RA and the units of Dec having the names "minutes" and "seconds", one can speak of "minutes (or seconds) *of time*" to distinguish RA measures from the "minutes of arc" used to measure Dec.

3.1.6 The relation between the equatorial and the horizon systems

Figure 3.7 shows the celestial sphere with some of the features of both the horizon and equatorial systems marked. The figure assumes an observer, "O", located at about 60 degrees north latitude on Earth. Note the altitude of

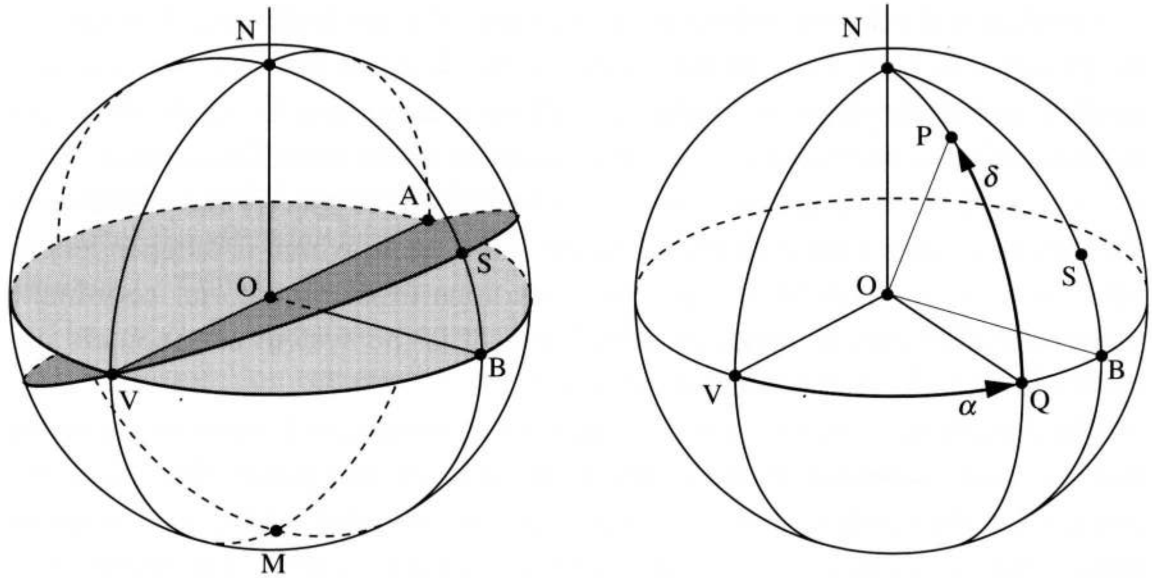


Fig. 3.6 The equatorial coordinate system. In both celestial spheres pictured, the equator is the great circle passing through points V and B, and the ecliptic is the great circle passing through points V and S. The left-hand sphere shows the locations of the north (N) and south (M) celestial poles, the vernal (V) and autumnal (A) equinoxes, the summer (S) solstice, and the hour circles for 0 Hr (arc NVM) and 6 Hr (arc NBM) of right ascension. The right-hand sphere shows the right ascension (∠VOQ, or α) and declination (∠QOP, or δ) of the point P.

the north celestial pole (angle NOP in Figure 3.7a). You should be able to construct a simple geometric argument to convince yourself that: the altitude angle of the north celestial pole equals the observer's geodetic latitude.

Observer "O," using the horizon system, will watch the celestial sphere turn, and see stars move along the projected circles of constant declination. Figure 3.7a shows the declination circle of a star that just touches the northern horizon. Stars north of this circle never set and are termed ***circumpolar***. Figure 3.7a also shows the declination circle that just touches the southern horizon circle, and otherwise lies entirely below it. Unless she changes her latitude, "O" can never see any of the stars south of this declination circle.

Reference to Figure 3.7a also helps define a few other terms. Stars that are neither circumpolar nor permanently below the horizon will rise in the east, cross, or ***transit***, the observer's celestial meridian, and set in the west. When a star transits the meridian it has reached its greatest altitude above the horizon, and is said to have reached its ***culmination***. Notice that circumpolar stars can be observed to cross the meridian twice each sidereal day (once when they are highest in the sky, and again when they are lowest). To avoid confusion, the observer's celestial meridian is divided into two pieces at the pole. The smaller bit visible between the pole and the horizon (arc NP in the figure) is called the lower meridian, and the remaining piece (arc PTML) is called the ***upper meridian***.

Figure 3.7b shows a star, S, which has crossed the upper meridian some time ago and is moving to set in the west. A line of constant right ascension is a great circle called an ***hour circle***, and the hour circle for star S is shown in the figure.

You can specify how far an object is from the meridian by giving its ***hour angle***. The hour circle of an object and the upper celestial meridian intersect at the pole. The hour angle, HA, is the angle between them. Application of the law of sines to a spherical right triangle shows that the hour angle could also be

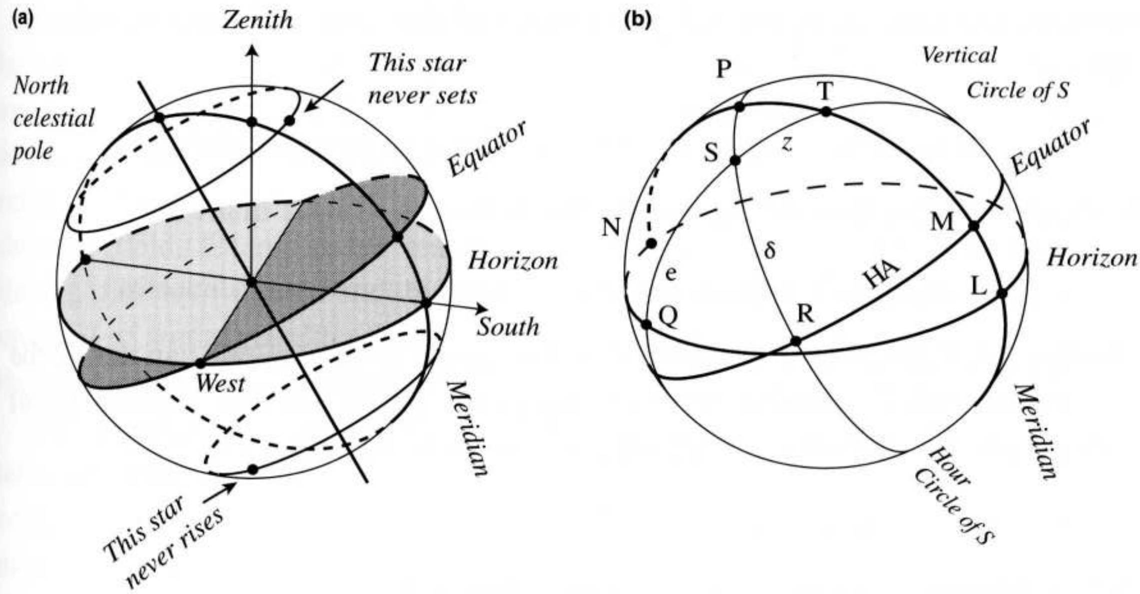


Fig. 3.7 The horizon and equatorial systems. Both spheres show the horizon, equator and observer's meridian, the north celestial pole at P, and the zenith at T. Sphere (a) illustrates the diurnal paths of a circumpolar star and of a star that never rises. Sphere (b) shows the hour circle (PSR) of a star at S, as well as its declination, δ, its hour angle, HA = arc RM = ∠MPS, its altitude, *e*, and its zenith distance, *z*.

measured along the equator, as the arc that runs from the intersection of the meridian and equator to the intersection of the star's hour circle and the equator (arc RM in Figure 3.7b). Hour angle, like right ascension, is usually measured in time units. Recalling that RA is measured in the plane of the equator, we can state one other definition of the hour angle:

$$\text{HA of the object} = \text{RA on meridian} - \text{RA of the object}$$

The hour angle of a star is useful because it tells how long ago (in the case of positive HA) or how long until (negative HA) the star crossed, or will cross, the upper meridian. The best time to observe an object is usually when it is highest in the sky, that is, when the HA is zero and the object is at culmination.

To compute the hour angle from the formula above, you realize that the RA of the object is always known – you can look it up in a catalog or read it from a star chart. How do you know the right ascension of objects on the meridian? You read that from a ***sidereal clock***.

A sidereal clock is based upon the apparent motions of the celestial sphere. A clockmaker creates a clock that ticks off exactly 24 uniform "sidereal" hours between successive upper meridian transits by the vernal equinox (a period of about 23.93 "normal" hours, remember). If one adjusts this clock so that it reads zero hours at precisely the moment the vernal equinox transits, then it gives the correct sidereal time.

$$\text{Sidereal day} = \text{Time between upper meridian transits by the vernal equinox}$$

A sidereal clock mimics the sky, where the hour circle of the vernal equinox can represent the single hand of a 24-hour clock, and the observer's meridian can represent the "zero hour" mark on the clockface. There is a nice correspondence between the reading of any sidereal clock and the right ascension coordinate, namely

$$\text{sidereal time} = \text{right ascension of an object on the upper meridian}$$

It should be clear that we can restate the definition of hour angle as:

$$\text{HA of object} = \text{sidereal time now} - \text{sidereal time object culminates}$$

If either the sidereal time or an object's hour angle is known, one can derive the coordinate transformations between equatorial (α, δ) and the horizon (e, a) coordinates for that object. Formulae are given in Appendix D.

3.1.7 Measuring equatorial coordinates.

Astronomers use the equatorial system because RA and Dec are easily determined with great precision from Earth-based observatories. You should have a general idea of how this is done. Consider a specialized instrument, called a ***transit telescope*** (or ***meridian circle***): the transit telescope is constrained to point only at objects on an observer's celestial meridian – it rotates on an axis aligned precisely east–west. The telescope is rigidly attached to a graduated circle centered on this axis. The circle lies in the plane of the meridian and rotates with the telescope. A fixed index, established using a plumb line perhaps, always points to the zenith. By observing where this index falls on the circle, the observer can thus determine the altitude angle (or zenith distance) at which the telescope is pointing. The observer is also equipped with a sidereal clock, which ticks off 24 sidereal hours between upper transits of the vernal equinox.

To use the transit telescope to determine declinations, first locate the celestial pole. Pick out a circumpolar star. Read the graduated circle when you observe the star cross the upper and then again when it crosses the lower meridian. The average of the two readings gives the location of $\pm 90°$ declination (the north or south celestial pole) on your circle. After this calibration you can then read the declination of any other transiting star directly from the circle.

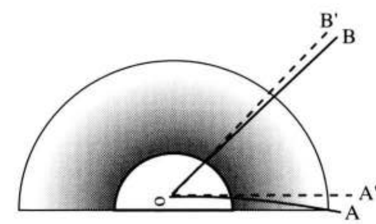
To find the *difference* in the RA of any two objects, subtract the sidereal clock reading when you observe the first object transit from the clock reading when you observe the second object transit. To locate the vernal equinox and the zero point for the RA coordinate, require that the right ascension of the Sun be zero when you observe its declination to be zero in the spring.

Astrometry is the branch of astronomy concerned with measuring the positions, and changes in position, of sources. Chapter 11 of Birney *et al.* (2006) gives a more though introduction to the subject than we will do here, and Monet (1988) gives a more advanced discussion. The Powerpoint presentation on the Gaia website (http://www.rssd.esa.int/Gaia) gives a good introduction to astrometry from space.

Observations with a transit telescope can measure arbitrarily large angles between sources, and the limits to the accuracy of ***large-angle astrometry*** are different from, and usually much more severe than, the limits to small-angle astrometry. In ***small-angle astrometry***, one measures positions of a source relative to a local reference frame (e.g. stars or galaxies) contained on the same detector field. Examples of small-angle astrometry are the measurement of the separation of double stars with a micrometer-equipped eyepiece, the measurement of stellar parallax from a series of photographs, or the measurement of the position of a minor planet in two successive digital images of the same field.

The angular size and regularity of the stellar images formed by the transit telescope limit the precision of large-angle astrometry. The astronomer or her computer must decide when and where the center of the image transits, a task made difficult if the image is faint, diffuse, irregular, or changing shape on a short time scale. In the optical or near infrared, atmospheric seeing usually limits ground-based position measurements to an accuracy of about 0.05 arcsec, or 50 milli-arcsec (mas).

Positional accuracy at radio wavelengths is much greater. The technique of ***very long baseline interferometry* (VLBI)** can determine coordinates for point-like radio sources (e.g. the centers of active galaxies) with uncertainties less than 1 mas. Unfortunately, most normal stars are not sufficiently powerful radio sources to be detected, and their positions must be determined by optical methods.



Zenith distance	90°	89°	85°	70°	50°	20°
Shift in arc min	35	24	10	2.6	1.7	0.3

Fig. 3.8 Atmospheric refraction. The observer is on the surface at point O. The actual path of a light ray from object A is curved by the atmosphere, and O receives light from direction A′. Likewise, the image of object B appears at B′ – a smaller shift in position because both the path length and the angle of incidence are smaller. Refraction thus reduces the zenith distance of all objects, affecting those close to the horizon more than those near the zenith. The table below the figure gives approximate shifts in arc minutes for different zenith distances.

There are other sources of error in wide-angle ground-based astrometry. Refraction by the atmosphere (see Figure 3.8 and Appendix D) changes the apparent positions of radio and (especially) optical sources. Variability of the atmosphere can produce inaccuracies in the correction made for refraction. Flexure of telescope and detector parts due to thermal expansion or variations in gravitational loading can cause serious systematic errors. Any change, for example, that moves the vertical index relative to the meridian circle will introduce inconsistencies in declination measurements.

Modern procedures for measuring equatorial coordinates are much more refined than those described at the beginning of this section, but the underlying principles are the same. Most ground-based transit measurements are automated with a variety of electronic image detectors and strategies for determining transit times.

Space-based large-angle astrometry uses principles similar to the ground-based programs. Although ground-based transit telescopes use the spinning Earth as a platform to define both direction and time scale, any uniformly spinning platform and any clock could be equivalently employed. The spin of the artificial satellite HIPPARCOS, for example, allowed it to measure stellar positions by timing transits in two optical telescopes mounted on the satellite. Because images in space are neither blurred by atmospheric seeing or subject to atmospheric refraction, most of the 120,000 stars in the HIPPARCOS catalog have positional accuracies around 0.7 mas in each coordinate. A future mission, Gaia (the European Space Agency expects launch in 2012), will use a similar strategy with vastly improved technology. Gaia anticipates positional accuracies on the order of 0.007 mas (= 7 μas) for bright stars and accuracies better than 0.3 mas for close to a billion objects brighter than $V = 20$.

Catalogs produced with large-angle astrometric methods like transit telescope observations or the Gaia and HIPPARCOS missions are usually called ***fundamental catalogs***.

It is important to realize that although the relative positions of most "fixed" stars on the celestial sphere normally do not change appreciably on time scales of a year or so, their equatorial coordinates *do* change by as much as 50 arcsec per year due to precession and other effects. Basically, the location of the celestial pole, equator, and equinox are always moving (see Section 3.1.8 below). This is an unfortunate inconvenience. Any measurement of RA and Dec made with a transit circle or other instrument must allow for these changes. What is normally done is to correct measurements to compute the coordinates that the celestial location *would* have at a certain date. Currently, the celestial equator and equinox for the year 2000 (usually written as J2000.0) are likely to be used.

You should also realize that even the *relative* positions of some stars, especially nearby stars, do change very slowly due to their actual motion in space relative to the Sun. This ***proper motion***, although small (a large proper motion would be a few arcsec per century), will cause a change in coordinates over time, and an accurate specification of coordinates must give the ***epoch*** (or date) for which they are valid. See Section 3.4.2 below.

3.1.8 Precession and nutation

Conservation of angular momentum might lead one to expect that the Earth's axis of rotation would maintain a fixed orientation with respect to the stars. However, the Earth has a non-spherical mass distribution, so it does experience gravitational torques from the Moon (primarily) and Sun. In addition to this lunisolar effect, the other planets produce much smaller torques. As a result of all these torques, the spin axis changes its orientation, and the celestial poles and equator change their positions with respect to the stars. This, of course, causes the RA and Dec of the stars to change with time.

This motion is generally separated into two components, a long-term general trend called ***precession***, and a short-term oscillatory motion called ***nutation***. Figure 3.9 illustrates precession: the north *ecliptic* pole remains fixed with respect to the distant background stars, while the north *celestial* pole (NCP) moves in a small circle whose center is at the ecliptic pole. The precessional circle has a radius equal to the average obliquity (around 23 degrees), with the NCP completing one circuit in about 26,000 years, moving at a very nearly – but

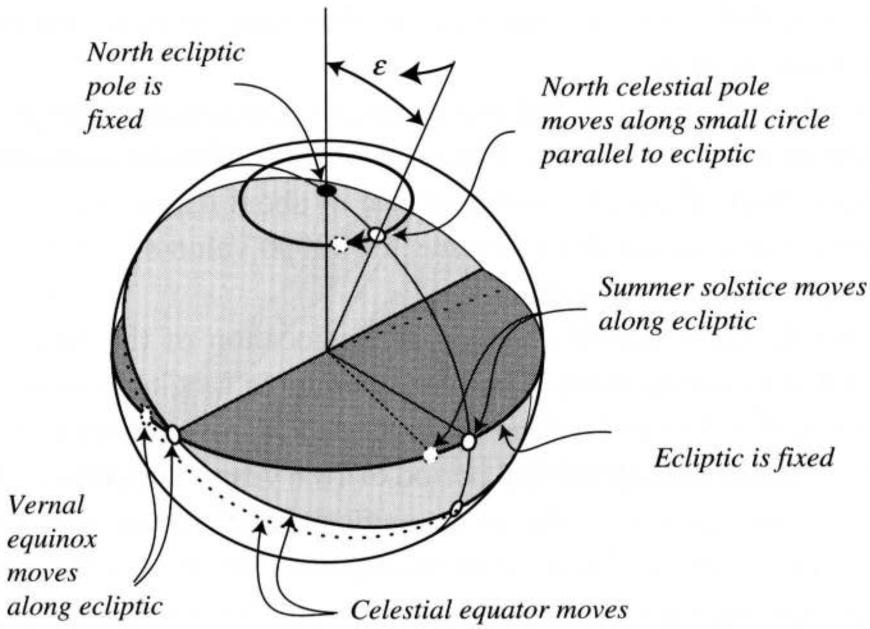


Fig. 3.9 Precession of the equinoxes. The location of the ecliptic and the ecliptic poles is fixed on the celestial sphere. The celestial equator moves so that the north celestial pole describes a small circle around the north ecliptic pole of radius equal to the mean obliquity.

not precisely – constant speed. The celestial equator, of course, moves along with the pole, and the vernal equinox, which is the fundamental direction for both the equatorial and ecliptic coordinate systems, moves westward along the ecliptic at the rate (in the year 2000) of 5029.097 arcsec (about 1.4 degrees) per century. Precession will in general cause both the right ascension and declination of every star to change over time, and will also cause the ecliptic longitude (but not the ecliptic latitude) to change as well.

The most influential ancient astronomer, Hipparchus of Rhodes (recorded observations 141–127 BCE) spectacularly combined the rich tradition of Babylonian astronomy, which was concerned with mathematical computation of future planetary positions from extensive historic records, and Greek astronomy, which focused on geometrical physical models that described celestial phenomena. He constructed the first quantitative geocentric models for the motion of the Sun and Moon, developed the trigonometry necessary for his theory, injected the Babylonian sexagesimal numbering system (360° in a circle) into western use, and compiled the first systematic star catalog. Hipparchus discovered lunisolar precessional motion, as a steady regression of the equinoxes, when he compared contemporary observations with the Babylonian records. Unfortunately, almost all his original writings are lost, and we know his work mainly though the admiring Ptolemy, who lived three centuries later.

Since the time of Hipparchus, the vernal equinox has moved about 30° along the ecliptic. In fact, we still refer to the vernal equinox as the "first point of Aries," as did Hipparchus, even though it has moved out of the constellation Aries and through almost the entire length of the constellation Pisces since his time. Precession also means that the star Polaris is only temporarily located near the north celestial pole. About 4500 years ago, at about the time the Egyptians constructed the Great Pyramid, the "North Star" was Thuban, the brightest star in Draco. In 12,000 years, the star Vega will be near the pole, and Polaris will have a declination of 43°.

Unlike lunisolar precession, planetary precession actually changes the angle between the equator and ecliptic. The result is an oscillation in the obliquity so that it ranges from 22° to 24°, with a period of about 41,000 years. At present, the obliquity is decreasing from an accepted J2000 value of 23° 26′ 21.4″ at a rate of about 47 arcsec per century.

Nutation, the short period changes in the location of the NCP, is usually separated into two components. The first, nutation in longitude, is an oscillation of the equinox ahead of and behind the precessional position, with an amplitude of about 9.21 arcsec and a principal period of 18.6 years. The second, nutation in obliquity, is a change in the value of the angle between the equator and ecliptic. This also is a smaller oscillation, with an amplitude of about 6.86 arcsec and an identical principal period. Both components were discovered telescopically by James Bradley (1693–1762), the third British Astronomer Royal.

3.1.9 Barycentric coordinates

Coordinates measured with a transit telescope from the surface of the moving Earth as described in the preceding section are in fact measured in a non-inertial reference frame, since the spin and orbital motions of the Earth accelerate the telescope. These ***apparent equatorial coordinates*** exhibit variations introduced by this non-inertial frame, and their exact values will depend on the time of observation and the location of the telescope. Catalogs therefore give positions in an equatorial system similar to the one defined as above, but whose origin is at the barycenter (center of mass) of the Solar System. Barycentric coordinates use the mean equinox of the catalog date (a fictitious equinox which moves with precessional motion, but not nutational). The barycentric coordinates are computed from the apparent coordinates by removing several effects. In addition to precession and nutation, we will discuss two others. The first, due to the changing vantage point of the telescope as the Earth executes its orbit, is called ***heliocentric stellar parallax***. The small variation in a nearby object's apparent coordinates due to parallax depends on the object's distance and is an important quantity described in Section 3.2.2.

The second effect, caused by the finite velocity of light, is called the ***aberration of starlight***, and produces a shift in every object's apparent coordinates. The magnitude of the shift depends only on the angle between the object's direction and the direction of the instantaneous velocity of the Earth. Figure 3.10 shows a telescope in the barycentric coordinate system, drawn so that the velocity of the telescope, at rest on the moving Earth, is in the $+x$ direction. A photon from a distant object enters the telescope at point A, travels at the speed of light, c, and exits at point B. In the barycentric frame, the photon's path makes an angle θ with the x-axis. However, if the photon is to

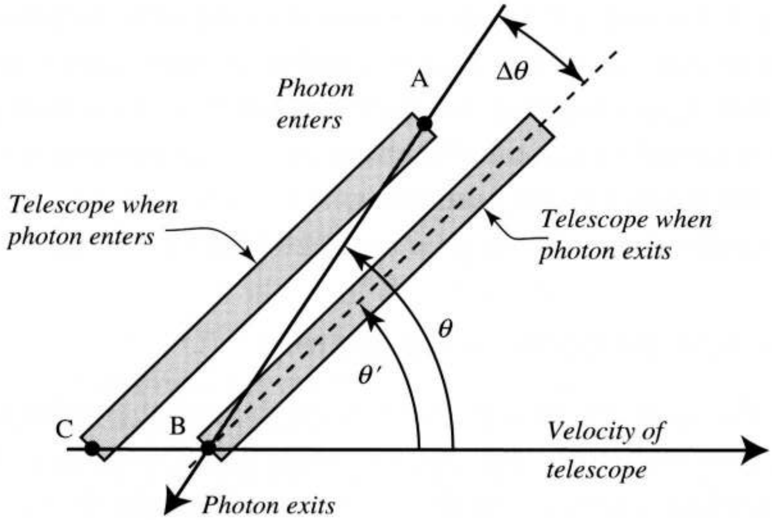


Fig. 3.10 The aberration of starlight. A telescope points towards a source. The diagram shows the telescope moving to the right in the barycentric frame. The apparent direction of the source, θ', depends on the direction and magnitude of the telescope velocity.

enter and exit the moving telescope successfully, the telescope must make an angle $\theta' = \theta - \Delta\theta$ with the x-axis in the frame fixed on the Earth. A little geometry shows that, if V is the speed of the Earth,

$$\Delta\theta = \frac{V}{c}\sin\theta$$

Thus aberration moves the apparent position of the source (the one measured by a telescope on the moving Earth) towards the x-axis. The magnitude of this effect is greatest when $\theta = 90°$, where it amounts to about 20.5 arcsec.

3.1.10 The ICRS

The International Astronomical Union (IAU) in 1991 recommended creation of a special coordinate system whose origin is at the barycenter of the Solar System, with a fundamental plane approximately coincident with the Earth's equatorial plane in epoch J2000.0. The x-axis of this ***International Celestial Reference System* (ICRS)** is taken to be in the direction of the vernal equinox on that date. However, unlike the equatorial system, or previous barycentric systems, the axes of the ICRS are defined and fixed in space by the positions of distant galaxies, not by the apparent motion of the Sun. Unlike Solar System objects or nearby stars, these distant objects have undetectable angular motions relative to one another. Their relative positions do not depend on our imperfect knowledge or observations of the Earth's rotation, precession, and nutation. Thus, the ICRS is a very good approximation of an inertial, non-rotating coordinate system.

In practice, radio-astronomical determinations of the equatorial coordinates of over 200 compact extragalactic sources (mostly quasars) define this inertial reference frame in an ongoing observing program coordinated by the International Earth Rotation Service in Paris. Directions of the ICRS axes are now specified to a precision of about 0.02 mas relative to this frame. The ICRS positions of optical sources are known primarily through HIPPARCOS and

Hubble Space Telescope (HST) observations near the optical counterparts of the defining radio sources, as well as a larger number of other radio sources. Approximately 100,000 stars measured by HIPPARCOS thus have ICRS coordinates known with uncertainties typical of that satellite's measurements, around 1 mas. Through the HIPPARCOS measurements, ICRS positions can be linked to the Earth-based fundamental catalog positions like FK5 (see Chapter 4).

3.1.11 The ecliptic coordinate system

The ecliptic, the apparent path of the Sun on the celestial sphere, can also be defined as the intersection of the Earth's orbital plane with the celestial sphere. The orbital angular momentum of the Earth is much greater than its spin angular momentum, and the nature of the torques acting on each system suggests that the orbital plane is far more likely to remain invariant in space than is the equatorial plane. Moreover, the ecliptic plane is virtually coincident with the plane of symmetry of the Solar System as well as lying nearly perpendicular to the Solar System's total angular momentum vector. As such, it is an important reference plane for observations and dynamical studies of Solar System objects.

Astronomers define a geocentric coordinate system in which the ecliptic is the fundamental plane and the vernal equinox is the fundamental direction. Measure ecliptic longitude, λ, from west to east in the fundamental plane. Measure the ecliptic latitude, β, positive northward from the ecliptic. Since the vernal equinox is also the fundamental direction of the equatorial system, the north ecliptic pole is located at RA = 18 hours and Dec = $90° - \varepsilon$, where ε is the obliquity of the ecliptic.

The ecliptic is so nearly an invariant plane in an inertial system that, unlike the equatorial coordinates, the ecliptic latitudes of distant stars or galaxies will *not* change with time because of precession and nutation. Ecliptic longitudes on the other hand, are tied to the location of the equinox, which is in turn defined by the spin of the Earth, so longitudes will have a precessional change of about 50″ per year.

3.1.12 The Galactic coordinate system

> Whoever turns his eye to the starry heavens on a clear night will perceive that band of light... designated by the name Milky Way... it is seen to occupy the direction of a great circle, and to pass in uninterrupted connection round the whole heavens:... so perceptibly different from the indefiniteness of chance, that attentive astronomers ought to have been thereby led, as a matter of course, to seek carefully for the explanation of such a phenomenon.
>
> – Immanuel Kant, *Universal Natural History and a Theory of the Heavens*, 1755

Kant's explanation for the Milky Way envisions our own Galaxy as a flattened system with approximately cylindrical symmetry composed of a large number of stars, each similar to the Sun. Astronomers are still adding detail to Kant's essentially correct vision: we know the Sun is offset from the center by a large fraction of the radius of the system, although the precise distance is uncertain by at least 5%. We know the Milky Way, if viewed from above the plane, would show spiral structure, but are uncertain of its precise form. Astronomers are currently investigating extensive evidence of remarkable activity in the central regions.

It is clear that the central plane of the disk-shaped Milky Way Galaxy is another reference plane of physical significance. Astronomers have specified a great circle (the ***Galactic plane***) that approximates the center-line of the Milky Way on the celestial sphere to constitute the fundamental plane of the Galactic coordinate system. We take the fundamental direction to be the direction of the center of the galaxy. Galactic latitude (b or b^{II}) is then measured positive north (the Galactic hemisphere contains the north celestial pole) of the plane, and Galactic longitude (l or l^{II}) is measured from Galactic center so as to constitute a right-handed system.

Since neither precession nor nutation affects the Galactic latitude and longitude, these coordinates would seem to constitute a superior system. However, it is difficult to measure l and b directly, so the Galactic coordinates of any object are in practice derived from its equatorial coordinates. The important parameters are that the north Galactic pole ($b = +90°$) is defined to be at

$$\alpha = 12{:}49{:}00, \delta = +27.4° \text{ (equator and equinox of 1950)}$$

and the Galactic center ($l = b = 0$) at

$$\alpha = 17{:}42{:}24, \delta = -28°55' \text{ (equator and equinox of 1950)}$$

3.1.13 Transformation of coordinates

Transformation of coordinates involves a combination of rotations and (sometimes) translations. Note that for very precise work, (the transformation of geocentric to ICRS coordinates, for example) some general-relativistic modeling may be needed.

Some of the more common transformations are addressed in the various national almanacs, and for systems related just by rotation (equatorial and Galactic, for example), you can work transformations out by using spherical trigonometry (see Section 3.1.2). Some important transformations are given in Appendix D, and calculators for most can be found on the Internet.

3.2 The third dimension

Determining the distance of almost any object in astronomy is notoriously difficult, and uncertainties in the coordinate r are usually enormous compared to uncertainties in direction. For example, the position of Alpha Centauri, the nearest star after the Sun, is uncertain in the ICRS by about 0.4 mas (three parts in 10^9 of a full circle), yet its distance, one of the best known, is uncertain by about one part in 2500. A more extreme example would be one of the quasars that define the ICRS, with a typical positional uncertainty of 0.02 mas (six parts in 10^{10}). Estimates of the distances to these objects depend on our understanding of the expansion and deceleration of the Universe, and are probably uncertain by at least 10%. This section deals with the first two rungs in what has been called the "cosmic distance ladder," the sequence of methods and calibrations that ultimately allow us to measure distances (perhaps "estimate distances" would be a better phrase) of the most remote objects.

3.2.1 The astronomical unit

We begin in our own Solar System. Kepler's third law gives the scale of planetary orbits:

$$a = P^{2/3}$$

where a is the average distance between the planet and the Sun measured in ***astronomical units*** (AU, or, preferably, au) and P is the orbital period in years. This law sets the *relative* sizes of planetary orbits. One au is defined to be the mean distance between the Earth and Sun, but the length of the au in meters, and the absolute scale of the Solar System, must be measured empirically.

Figure 3.11 illustrates one method for calibrating the au. The figure shows the Earth and the planet Venus when they are in a position such that apparent angular separation between Venus and the Sun, as seen from Earth, (the *elongation* of Venus) is at a maximum. At this moment, a radio (radar) pulse is sent from the Earth towards Venus, and a reflected pulse returns after elapsed time Δt. The Earth-to-Venus distance is just

$$\frac{1}{2}c\Delta t$$

Thus, from the right triangle in the figure, the length of the line ES is one au or

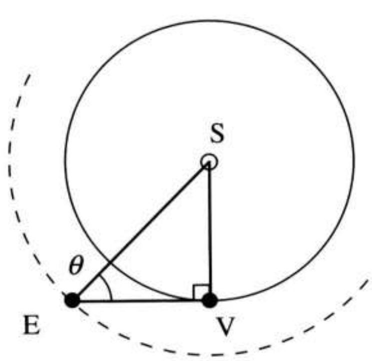


Fig. 3.11 Radar ranging to Venus. The astronomical unit is the length of the line ES, which scales with EV, the Earth-to-Venus distance.

$$1\ \text{au} = \frac{c\Delta t}{2\cos\theta}$$

Clearly, some corrections need to be made because the orbit of neither planet is a perfect circle, but the geometry is known rather precisely. Spacecraft in orbit around Venus and other planets (Mars, Jupiter, and Saturn) also provide the opportunity to measure light-travel times, and similar geometric analyses yield absolute orbit sizes. The presently accepted value for the length of the au is

$$1\ \text{au} = 1.49\,5978 \times 10^{11}\ \text{m}$$

with an uncertainty of 1 part in 10^6.

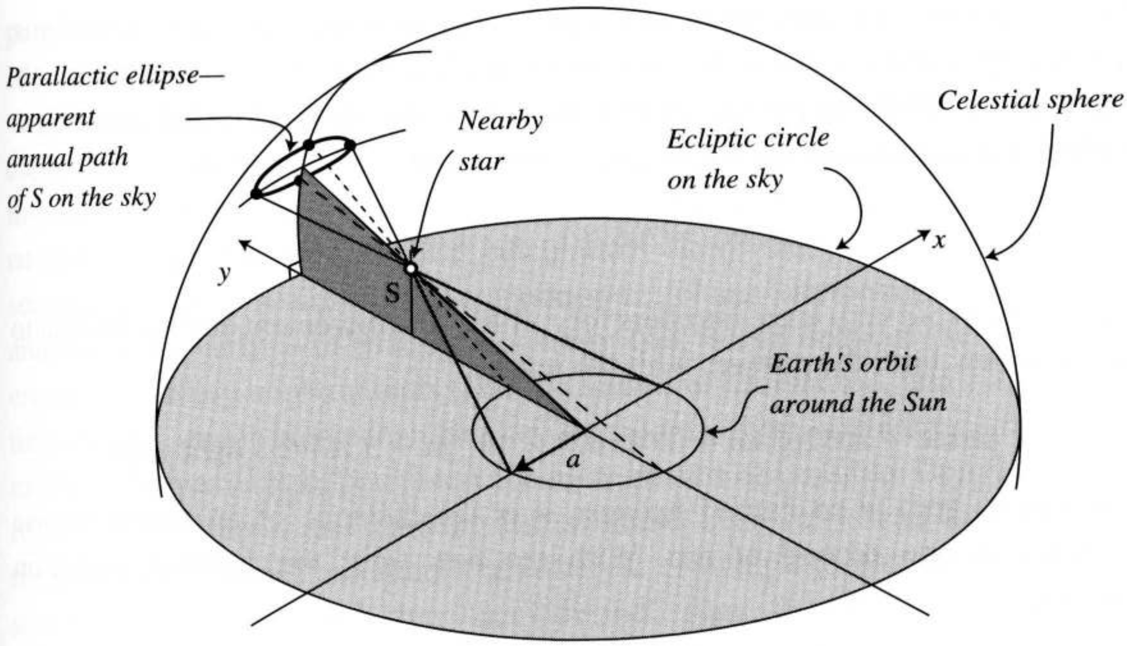


Fig. 3.12 The parallactic ellipse. The apparent position of the nearby star, S, as seen from Earth, traces out an elliptical path on the very distant celestial sphere as a result of the Earth's orbital motion.

3.2.2 Stellar parallax

Once the length of the au has been established, we can determine the distances to nearby stars through observations of ***heliocentric stellar parallax***. Figure 3.12 depicts the orbit of the Earth around the Sun. The plane of the orbit is the ecliptic plane, and we set up a Sun-centered coordinate system with the ecliptic as the fundamental plane, the z-axis pointing towards the ecliptic pole, and the y-axis chosen so that a nearby star, S, is in the y–z plane. The distance from the Sun to S is r. As the Earth travels in its orbit, the apparent position of the nearby star shifts in relation to very distant objects. Compared to the background objects, the nearby star appears to move around the perimeter of the ***parallactic ellipse***, reflecting the Earth's orbital motion.

Figure 3.13 shows the plane that contains the x-axis and the star. The parallax angle, p, is half the total angular shift in the star's position (the semi-major axis of the parallactic ellipse in angular units). From the right triangle formed by the Sun–star–Earth:

$$\tan p = \frac{a}{r}$$

where a is one au. Since p is in every case going to be very small, we make the small angle approximation: for $p \ll 1$:

$$\tan p \cong \sin p \cong p$$

So that for any right triangle where p is small:

$$p = \frac{a}{r}$$

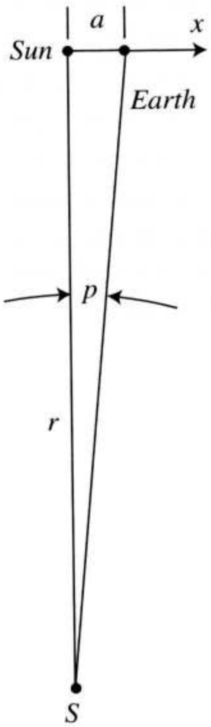


Fig. 3.13 The parallax angle.

In this equation, it is understood that a and r are measured in the same units (aus, for example) and p is measured in radians. Radian measure is somewhat inconvenient for small angles, so, noting that there are about 206 265 arcsec per radian, we can rewrite the small-angle formula as

$$p[\text{arcsec}] = 206\,265\frac{a}{r}\,[a, r \text{ in same units}]$$

Finally, to avoid very large numbers for r, it is both convenient and traditional to define a new unit, the ***parsec***, with the length:

$$1 \text{ parsec} = 206\,265 \text{ au} = 3.085\,678 \times 10^{16}\,\text{m} = 3.261\,633 \text{ light years}$$

The parsec (pc) is so named because it is the distance of an object whose *par*allax is one *sec*ond of arc. With the new unit, the parallax equation becomes:

$$p[\text{arcsec}] = \frac{a[\text{au}]}{r[\text{pc}]} \tag{3.1}$$

This equation represents a fundamental relationship between the small angle and the sides of the ***astronomical triangle*** (any right triangle with one very short side). For example, suppose a supergiant star is 20 pc away, and we measure its angular diameter with the technique of speckle interferometry as 0.023 arcsec. Then the physical diameter of of the star, which is the short side of the relevant astronomical triangle (the quantity a in Equation (3.1)), must be 20 × 0.023 pc arcsec = 0.46 au.

In the case of stellar parallax, the short side of the triangle is always 1 au. If $a = 1$ in Equation (3.1), we have:

$$p[\text{arcsec}] = \frac{1}{r[\text{pc}]} \tag{3.2}$$

In the literature, the parallax angle is often symbolized as π instead of p. Note that the parallactic ellipse will have a semi-major axis equal to p, and a semi-minor axis equal to $p \sin \lambda$, where λ is the ecliptic latitude of the star. The axes of an ellipse fit to multiple observations of the position of a nearby star will therefore estimate its parallax.

There are, of course, uncertainties in the measurement of small angles like the parallax angle. Images of stars formed by Earth-based telescopes are typically blurred by the atmosphere, and are seldom smaller than a half arc second in diameter, and are often much larger. In the early days of telescopic astronomy, a great visual observer, James Bradley (1693–1762), like many astronomers before him, undertook the task of measuring stellar parallax. Bradley could measure stellar positions with a precision of about 0.5 arcsec (500 milliarcseconds or mas). This precision was sufficient to discover the phenomena of nutation and aberration, but not to detect a stellar parallax (the largest parallax of a bright star visible at Bradley's latitude is that of Sirius, i.e. 379 mas).

A few generations later, Friedrich Wilhelm Bessel (1784–1846), a young clerk in an importer's office in Bremen, began to study navigation in order to move ahead in the business world. Instead of mercantile success, Bessel discovered his love of astronomical calculation. He revised his career plans and secured a post as assistant to the astronomer Johann Hieronymous Schroeter, and began an analysis of Bradley's observations. Bessel deduced the systematic errors in Bradley's instruments (about 4 arcsec in declination, and 1 second of time in RA – much worse than Bradley's random errors), and demonstrated that major improvements in positional accuracy should be possible. Convinced that reducing systematic and random errors would eventually lead to a parallax measurement, Bessel mounted a near-heroic campaign to monitor the double star 61 Cygni along with two "background" stars. In 1838, after a 25-year effort, he succeeded in measuring the parallax with a precission that satisfied him. (His value for the parallax, 320 mas, is close to the modern value of 286 mas.) Bessel's labor was typical of his ambition[2] and meticulous attention to error reduction. The 61 Cygni parallax project was Herculean, and parallaxes for any but the very nearest stars emerged only after the introduction of photography.[3]

Beginning in the late 1880s, photography gradually transformed the practice of astronomy by providing an objective record of images and by allowing the accumulation of many photons in a long exposure. With photography, human eyesight no longer limited human ability to detect faint objects with a telescope. Photography vastly augmented the power of small-angle astrometry. Astronomical photographs (negatives) were usually recorded on emulsion-covered glass plates at the telescope, then developed in a darkroom.

Away from the telescope, astronomers could then measure the positions of objects on the plate, usually with ***a plate-measuring machine*** (Figure 3.14). During the twentieth century, such machines became increasingly automated,

[2] Bessel pioneered mathematical analysis using the functions that now bear his name. He spent 30 years measuring the "Prussian degree" – the length, in meters, of a degree of arc of geodetic latitude. This was part of an international effort to determine the shape of the Earth from astronomical measurements. After his publication of the corrected positions of the 3222 stars in Bradley's catalog, Bessel went on to measure his own positions for 62,000 other stars, and inspired his student, F.W. Argelander, to organize a project to determine the transit-circle positions for all stars brighter than ninth magnitude in the northern hemisphere – about a third of a million objects.

[3] Wilhelm Struve published his measurement of the parallax of Vega immediately after Bessel's publication (he had actually completed his analysis before Bessel had) and Thomas Henderson published the parallax of Alpha Centauri in the next year. Bessel's measurement was the most accurate.

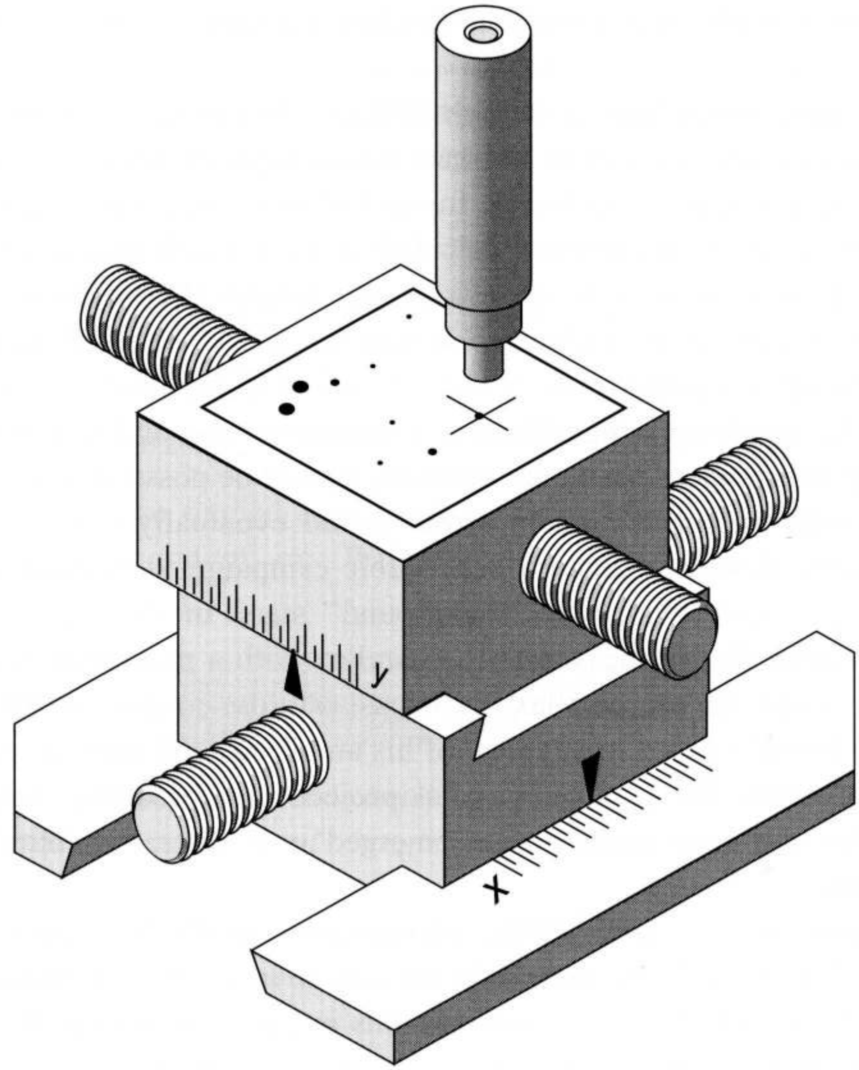


Fig. 3.14 Schematic of a measuring engine. A fixed microscope views an object on a photographic plate. The (*x*, *y*) position of the plate is controlled by two precision screws, and its value can be read from scales on the moving stages.

precise, and expensive. Computer-controlled measuring machines called ***microdensitometers***, which record the darkness of the image at each position on a plate, became important astronomical facilities. Direct digital recording of images with electronic arrays, beginning in the 1970s, gradually made the measuring machine unnecessary for many observations. Nevertheless, photography is still employed in some vital areas of astronomy, and measuring machines still continue to produce important data.

Conventional modern small-angle astrometry, using photographic or electronic array detectors, can measure the relative position of a nearby star with a precision of the something like 50 mas in a single measurement. This uncertainty can be reduced by special equipment and techniques, and by repeated measurements as a star shifts around its parallactic ellipse. One's ability to measure a parallax depends on the presence of suitable background objects and the stability of the observing system over several years. Uncertainty in p from conventional ground-based small-angle astrometry can be routinely reduced to around 5 mas with repeated measurements (50 observations of a single star are not unusual). Even so, this means that only parallaxes larger than 50 mas will have uncertainties smaller than 10%, so only those stars nearer than $1/0.05 = 20$ pc can be considered to have distances precisely known by the ground-based parallax method. There are approximately 1000 stars detected closer than 20 pc, a rather small number compared to the 10^{11} or so stars in the Milky Way. Appendix D1 lists the nearest stars, based upon the best current parallaxes.

Ground-based parallax measurements can approach 0.5 mas precision with suitable technique and special instruments, at least for a small number of the best-studied stars. Nevertheless, spaced-based methods have produced the greatest volume of precision measurements. The HIPPARCOS space mission measured the positions, parallaxes, and proper motions of 117,955 stars in the four years between 1989 and 1993. The median precision of the parallaxes is 0.97 mas for stars brighter than $V = 8.0$. The Hubble Space Telescope has made a much smaller number of measurements of similar accuracy. The planned Gaia mission anticipates precisions of 7 mas for $V = 7$, 10–25 μas for $V = 15$, and around 500 μas at $V = 20$. The Space Interferometry Mission (SIM lite) spacecraft, with a planned launch sometime after 2015, would use a new observing method (a Michaelson interferometer) to achieve precisions of 4 μas for stars brighter than $V = 15$, and of 12 μas at $V = 20$.

3.3 Time

> Alice sighed wearily. "I think you might do something better with the time," she said, "than wasting it in asking riddles that have no answers."
> "If you knew Time as well as I do," said the Hatter," you wouldn't talk about wasting IT. It's HIM. . . . I dare say you never even spoke to Time!"
> "Perhaps not," Alice cautiously replied; "but I know I have to beat time when I learn music."
>
> – Lewis Carroll, *Alice's Adventures in Wonderland*, 1897

Time is a physical quantity of which we have never enough, save for when we have too much and it gets on our hands. Ambition to understand its nature has consumed the time of many. It is unclear how much of it has thereby been wasted in asking riddles with no answers. Perhaps time will tell.

3.3.1 Atomic time

Measuring time is a lot easier than understanding it. The way to measure time is to "beat" it, like Alice. In grammar school, I learned to count seconds by pronouncing syllables: "Mississippi one, Mississippi two, Mississippi three. . . ." A second of time is thus, roughly, the duration required to enunciate five syllables. A similar definition, this one set by international agreement, invokes a more objective counting operation:

> **1 second** (Système International, or SI second) = the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.

A device that counts the crests of a light wave and keeps a continuous total of the elapsed SI seconds is an ***atomic clock***. An atomic clock located at rest on the surface of the Earth keeps **TAI** or ***international atomic time*** (TAI = *Temps Atomique International*). Practical atomic clocks have a precision of about 2 parts in 10^{13}. International atomic time is the basis for dynamical computations involving time as a physical parameter and for recording observations made on the surface of the Earth. Things get a little complicated if you compare an atomic clock on the surface of the Earth with one located elsewhere (like the barycenter of the Solar System). Relativity theory and observation show that such clocks will not necessarily run at the same rate, but, when compared, will differ according to their relative velocities (special relativity) and according to their accelerations or local gravitational fields (general relativity). Precise time-keeping accounts for relativity effects, but the starting time scale in these computations is generally TAI.

The ***astronomical day*** is defined as 86,400 SI seconds. There are, however, other kinds of days.

3.3.2 Solar time

Early timekeepers found it most useful and natural to count days, months, and years, and to subdivide these units. The day is the most obvious and practical of these units since it correlates with the light–dark cycle on the surface of the Earth. Much of early timekeeping was a matter of counting, grouping, and subdividing days. Since the rotation of the Earth establishes the length of the day, counting days is equivalent to counting rotations.

Figure 3.15 illustrates an imaginary scheme for counting and subdividing days. The view is of the Solar System, looking down from above the Earth's north pole, which is point P. The plane of the page is the Earth's equatorial plane, and the large circle represents the equator itself. The small circle represents the position of the Sun projected onto the equatorial plane. In the figure, we assume the Sun is motionless, and we attach hour markers just outside the equator as if we were painting the face on an Earth-sized 24-hour clock. These markers are motionless as well, and are labeled so that they increase counter-clockwise, and so that the marker in the direction of the Sun is labeled 12, the one opposite the Sun, 0. The choice of 24 hours around the circle, as well as the subdivision into 60 minutes per hour and 60 seconds per minute, originated with the ancient Babylonian sexagesimal (base 60) number system.

Point O in the figure is the location of a terrestrial observer projected onto the equatorial plane. This observer's meridian projects as a straight line passing through O and P. The figure extends the projected meridian as an arrow, like the hand of a clock, which will sweep around the face with the painted numbers as the Earth rotates relative to the Sun. Since we are using the Sun as the reference marker, this turning of the Earth is actually a combination of spin and orbital

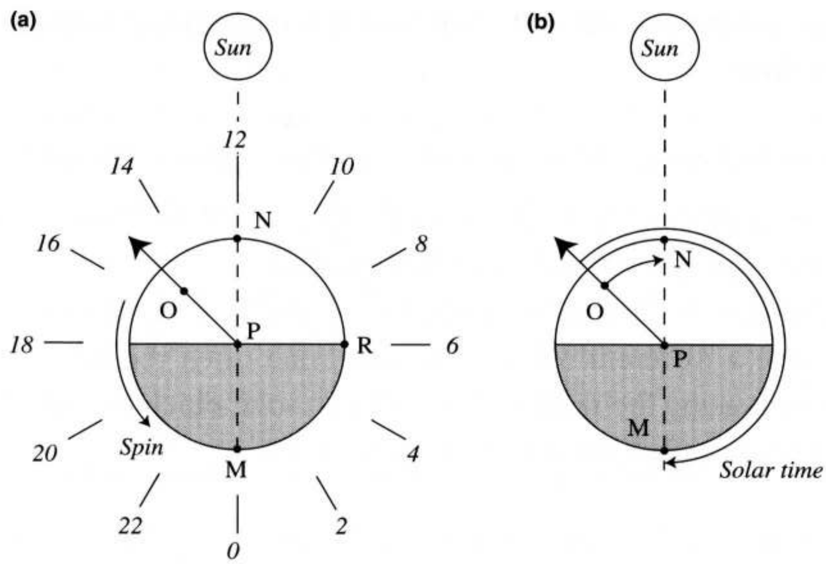


Fig. 3.15 A view of the equatorial plane from above the Earth's north pole, P. (a) The meridian of the observer points to the apparent solar time. (b) The apparent solar time, ∠OPM, equals the hour angle of the Sun, ∠OPN, plus 12 hours.

on. The meridian points to the number we will call the ***local apparent solar*** Each cycle of the meridian past the zero mark (midnight) starts a new day ne observer. Every longitude has a different meridian and thus a different time. The local solar time, for example, is 12 hours for an observer at point the figure, and 6 hours for an observer at R.

little consideration of Figure 3.15b should convince you of the following ition:

cal apparent solar time = the hour angle of the Sun as it appears on the sky (∠OPN), plus 12 hours

le observations (for example, with a sundial) will yield the local apparent time, but this method of timekeeping has a serious deficiency. Compared AI, local apparent solar time is non-uniform, mainly because of the Earth's al motion. Because of the obliquity of the ecliptic, motion in the orbit has a er east–west component at the solstices than at the equinoxes. In addition, use Earth's orbit is elliptical, Earth's orbital speed varies; it is greatest when closest to the Sun (at perihelion, around January 4) and slowest when est away (aphelion). As a result, apparent solar days throughout the year different lengths compared to the defined astronomical day of 86,400 SI nds.

his non-uniformity is troublesome for precise timekeeping. To remove it, trategy is to average out the variations by introducing the idea of the ***mean*** a fictitious body that moves along the celestial equator at uniform angular d, completing one circuit in one tropical year (i.e. equinox to equinox). If edefine the "Sun" in Figure 3.15 as the mean Sun, we can define a more rm time scale:

cal mean solar time = the hour angle of the fictitious mean Sun, plus 12 hours

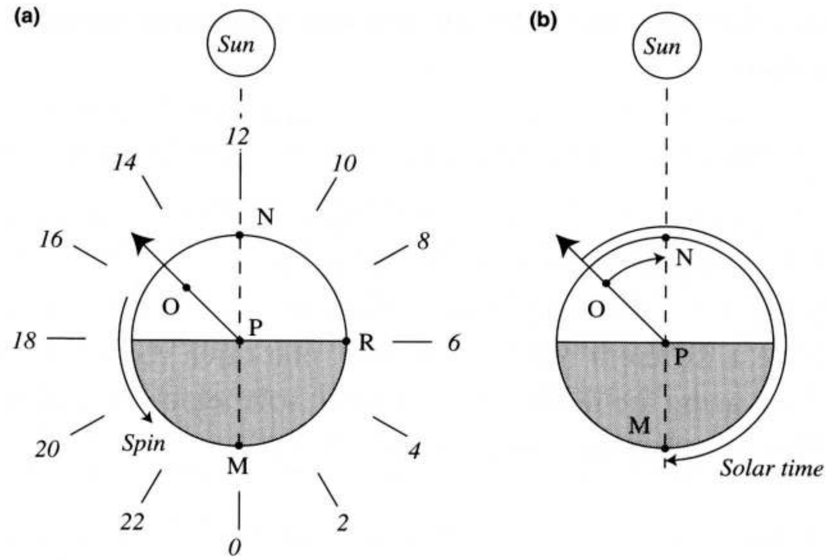


Fig. 3.15 A view of the equatorial plane from above the Earth's north pole, P. (a) The meridian of the observer points to the apparent solar time. (b) The apparent solar time, ∠OPM, equals the hour angle of the Sun, ∠OPN, plus 12 hours.

motion. The meridian points to the number we will call the ***local apparent solar time***. Each cycle of the meridian past the zero mark (midnight) starts a new day for the observer. Every longitude has a different meridian and thus a different solar time. The local solar time, for example, is 12 hours for an observer at point N in the figure, and 6 hours for an observer at R.

A little consideration of Figure 3.15b should convince you of the following definition:

local apparent solar time = the hour angle of the Sun as it appears on the sky (∠OPN), plus 12 hours

Simple observations (for example, with a sundial) will yield the local apparent solar time, but this method of timekeeping has a serious deficiency. Compared to TAI, local apparent solar time is non-uniform, mainly because of the Earth's orbital motion. Because of the obliquity of the ecliptic, motion in the orbit has a greater east–west component at the solstices than at the equinoxes. In addition, because Earth's orbit is elliptical, Earth's orbital speed varies; it is greatest when it is closest to the Sun (at perihelion, around January 4) and slowest when furthest away (aphelion). As a result, apparent solar days throughout the year have different lengths compared to the defined astronomical day of 86,400 SI seconds.

This non-uniformity is troublesome for precise timekeeping. To remove it, one strategy is to average out the variations by introducing the idea of the ***mean Sun***: a fictitious body that moves along the celestial equator at uniform angular speed, completing one circuit in one tropical year (i.e. equinox to equinox). If we redefine the "Sun" in Figure 3.15 as the mean Sun, we can define a more uniform time scale:

local mean solar time = the hour angle of the fictitious mean Sun, plus 12 hours

The difference between the apparent and the mean solar times is called the ***equation of time***:

$$\text{equation of time} = \text{local apparent solar time} - \text{local mean solar time}$$

The equation of time takes on values in the range ± 15 minutes in the course of a year. See Appendix D for more information.

To circumvent the difficulty arising from the fact that every longitude on Earth will have a different mean solar time, one often records or predicts the time of an event using the reading from a mean solar clock located at the zero of longitude. This is called the universal time (UT):

$$\text{Universal time (UT or UT1)} = \text{mean solar time at Greenwich}$$

The UT clock, of course, is actually located in your laboratory – it is simply set to agree with the mean solar time at the longitude of Greenwich. Thus, if the Moon were to explode, everyone on Earth would agree about the UT of the mishap, but only people at the same longitude would agree about the mean solar time at which it occurs.

Although a big improvement on apparent solar time, the UT1 is still not completely uniform. For one thing, the precession rate (needed to compute the mean Sun) is imperfectly known and changes over long time scales. The major difficulty, however, is that the spin of the Earth is not quite uniform. The largest variations are due to varying tidal effects that have monthly and half-monthly periods, as well as seasonal (yearly) variations probably due to thermal and meteorological effects. A smaller, random variation, with a time scale of decades, is probably due to poorly understood core–mantle interactions. Finally, over the very long term, tidal friction causes a steady slowing of the spin of the Earth. As result of this long-term trend, the mean solar day is getting longer (as measured in SI seconds) at the rate of about 0.0015 seconds per century. Thus, on the time scale of centuries, a day on the UT1 clock, (the mean solar day) is increasing in duration compared to the constant astronomical day of 86,400 SI seconds, and is fluctuating in length by small amounts on shorter time scales.

The International Earth Rotation Service (IERS), in Paris, has taken the monitoring of the variations in the terrestrial spin rate as one of its missions. In order to coordinate the Earth's rotation with TAI, the US Naval Observatory, working for the IERS, maintains the ***coordinated universal time*** (UTC) clock. Coordinated universal time approximates UT1, but uses SI seconds as its basic unit. To keep pace with UT1 to within a second, the UTC clock introduces an integral number of "leap" seconds as needed. Because of the random component in the acceleration of the Earth's spin, it is not possible to know in advance when it will be necessary to add (or remove) a leap second. A total of 22 leap seconds were counted by TAI (but not by UTC) between 1972 and the end of 1998. Coordinated universal time is the basis for most legal time systems (zone time).

Unlike UT or UTC, local solar time at least has the practical advantage of approximate coordination with local daylight: at 12 noon on the local mean solar clock, you can assume the Sun is near the meridian. However, every longitude will have a different meridian and a different local solar time. Even nearby points will use different clocks. To deal in a practical fashion with the change in mean solar time with longitude, most legal clocks keep zone time:

$$\text{zone time} = \text{UTC} + \text{longitude correction for the zone}$$

This strategy ensures that the legal time is the same everywhere inside the zone. Zones are usually about 15° wide in longitude, so the longitude correction is usually an integral number of hours. (Remember the Earth spins at a rate of 15 degrees per hour.) For example, Eastern Standard Time (longitude 75°) = UTC – 5 hours, Pacific Standard Time (longitude 120°) = UTC – 8 hours.

Time services provide signals for setting clocks to the current UTC value. In the USA, the National Institute of Standards and Technology broadcasts a radio signal "(stations WWV and WWVH) at 2.5, 5, 10, 15 and 20 MHz that contain time announcements and related information, and at 60 kHz (station WWVB) for synchronization of clocks. Computer networks can synchronize to UTC using standard protocols (ITS and ACTS). A convenient one-time check on UTC is at the US Naval Observatory website (http://www.usno.navy.mil/USNO), which is also a good source for details about various times scales and terrestrial coordinate systems.

Sidereal time is also defined by the rotation of the Earth and its precessional variations, and therefore does not flow uniformly, but follows the variations manifest in UT1:

$$\text{sidereal time} = \text{the hour angle of the mean vernal equinox of date}$$

Having defined the day, astronomers find it useful to maintain a continuous count of them:

$$\text{Julian date} = \text{number of elapsed UT or UTC days since} \\ \text{4713 BC January 1.5 (12 hrs UT on January 1.)}$$

It is also common to use a Julian date (JD), rather than a UT date, to specify the date. The date of the equator and equinox in a catalog of equatorial coordinates might be specified as

$$\text{J2000.0} = \text{"Julian epoch 2000.0"} = \text{2000 Jan 1.5 UT} = \text{JD 2451545.0}$$

Appendix A summarizes some other time units.

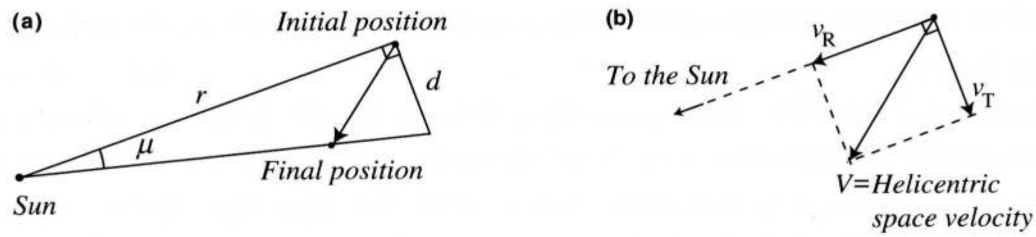


Fig. 3.16 Displacement in space and space velocity: (a) illustrates the relation between proper motion, μ, and the displacement in a unit time; (b) shows the two components of the space velocity.

3.4 Motion

3.4.1 Space motion

Consider an object that moves relative to the Sun. Figure 3.16, which is drawn in a three-dimensional coordinate system centered at the barycenter of the Solar System, shows the motion, that is, the displacement, of such an object over a suitably long time. The plane of the figure contains both the origin of coordinates and the displacement vector. Part (a) of the figure shows the actual displacement, while part (b) shows the displacement divided by the time interval, that is, the velocity. Both displacement and velocity vectors can be decomposed into radial and tangential components. The total velocity, usually called the ***space velocity***, is the vector sum of the ***tangential velocity*** and ***the radial velocity***:

$$\vec{V} = \vec{v}_{\mathrm{T}} + \vec{v}_{\mathrm{R}}$$
$$V = \sqrt{v_{\mathrm{T}}^2 + v_{\mathrm{R}}^2}$$

Measuring the two components requires two very different observing strategies. Astronomers can measure radial velocity directly with a spectrograph, and can measure tangential velocities indirectly by observing changes in position.

3.4.2 Proper motion

Suppose you have some quality observing time tonight, and you measure the position of a certain star in ICRS coordinates. Suppose, also, that 10 years in the future you observe the same star a second time. If this star were truly motionless with respect to the center of the Solar System and the distant galaxies that define the axes of the ICRS, then the coordinates you measure 10 years from now will be the same as those you measure tonight. Remember, effects like precession and parallax are not present in the ICRS.

On the other hand, most stars *do* move with respect to the ICRS axes. Especially if the star is nearby, its coordinates may very well change after only 10 years. The ***rate of change*** in coordinates is called the ***proper motion*** of the object. As the name suggests, proper motion reflects the "true" motion of the star with respect to the barycenter of the Solar System, and does not include those coordinate changes like aberration, precession, nutation, or heliocentric parallax that result from terrestrial motions. Proper motion, of course, is relative, and it may not be possible (or meaningful) to decide if it is due to motion of the barycenter or of the star.

Think about the objects that will *not* exhibit a proper motion over your 10-year observing interval. Certainly, these will include very distant objects like quasars, since they define the coordinate system. Also, any nearby star that has no tangential velocity will have no proper motion. Finally, you will not detect a proper motion for any object that is so far away that it does not change its angular position by an amount detectable by your instruments, even though its tangential velocity might be substantial.

The basic methods for measuring proper motion are fairly easy to understand. First some terminology: in astrometry, the ***epoch*** of an observation means the time of observation. The equatorial coordinate system used to record an observation also has a time associated with it, which is the date(s) of the ***equator and equinox*** employed. The two dates need not be the same. Unfortunately, even astronomers are not careful with this terminology, and will occasionally say "epoch" when they really mean "equator and equinox".

Keeping the above distinction in mind, you could determine the proper motion of a star by comparing its positions in fundamental catalogs for two different epochs (dates of observation), being careful to express the coordinates using the same barycentric equator and equinox. For example, look up the position of your star in a catalog for epoch 1934, which lists coordinates using the 1950 equator and equinox. Then find the same star in a second catalog, which gives its epoch 1994 position. The second catalog uses the equator and equinox of J2000. Now you must transform the epoch 1934 coordinates so that they are given in equator and equinox 2000 coordinates. Now that both positions are expressed in the same coordinate system (J2000), compute the difference between the 1994 position and the 1934 position. The difference, divided by the time interval (60 years, in this case) is the proper motion. Proper motions determined in this fashion are often called ***fundamental proper motions***. The method depends on astronomers doing the hard work of assembling at least two fundamental catalogs.

You can also measure proper motions using small-angle astrometry. Compare a photograph of a star field taken in 1994 with one taken with the same instrument in 1934. Align the photographs so that most of the images coincide, especially the faint background stars and galaxies. Any object that has shifted its position with respect to these "background objects" is exhibiting ***relative proper motion***. The possibility that there might be some net proper motion in the background objects limits the accuracy of this sort of measurement, as does the likelihood of changes in the instrument over a 60-year span. Nevertheless, relative proper motions are more easily determined than fundamental motions, and are therefore very valuable because they are available for many more stars.

You can, of course, use observations from different instruments (an old photograph and a recent CCD frame for example) to measure relative proper motions, but the analysis becomes a bit more complex and prone to systematic error.

Proper motion, represented by the symbol μ, is usually expressed in units of seconds of arc per year, or sometimes in seconds of arc per century. Since μ is a vector quantity, proper motion is generally tabulated as its RA and Dec components, μ_α and μ_δ.

The tangential component of the space velocity is responsible for proper motion. For the same tangential speed, nearer objects have larger proper motions. Refer to Figure 3.16. If an object at distance r has a tangential displacement $d = v_T t$ in time t, then, for small μ,

$$\mu = \frac{d/t}{r} = \frac{v_T}{r} \tag{3.3}$$

The statistical implications of Equation (3.3) are so important they are expressed in an astronomical "proverb": ***swiftness means nearness***. That is, given a group of objects with some distribution of tangential velocities, the objects with the largest values for μ (swiftness) will tend, statistically, to have the smallest values for r (nearness). Putting the quantities in Equation (3.3) in their usual units (km s^{-1} for velocity, parsecs for distance, seconds of arc per year for μ), it becomes

$$\mu = \frac{v_T}{4.74r}$$

This means, of course, that you can compute the tangential velocity if you observe both the proper motion and the parallax (p):

$$v_T = 4.74\frac{\mu}{p}$$

3.4.3 Radial velocity

On May 25, 1842, Christian Doppler (1803–1853) delivered a lecture to the Royal Bohemian Scientific Society in Prague. Doppler considered the situation in which an observer and a wave source are in motion relative to one another. He made the analogy between the behavior of both water and sound waves on the one hand, and of light waves on the other. Doppler correctly suggested that, in all three cases, the observer would measure a frequency or wavelength change that depended on the radial velocity of the source. The formula that expresses his argument is exact for the case of light waves from sources with small velocities:

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0} = \frac{v_R}{c} \tag{3.4}$$

Here λ_0 is the wavelength observed when the source is motionless, λ is the wavelength observed when the source has radial velocity v_R, and c is the speed of light. In his lecture, Doppler speculated that the differing radial velocities of stars were largely responsible for their different colors. To reach this conclusion, he assumed that many stars move at a considerable fraction of the speed of light relative to the Sun. This is wrong. But even though he was incorrect about the colors of the stars, the ***Doppler effect***, as expressed in Equation (3.4), was soon verified experimentally, and is the basis for all astronomical direct measurements of radial velocity. It is interesting to note that first Arnand Fizeau, in Paris in 1848, and then Ernst Mach, in Vienna in 1860, each independently worked out the theory of the Doppler effect without knowledge of the 1842 lecture.

Fizeau and Mach made it clear to astronomers how to *measure* a radial velocity. The idea is to observe a known absorption or emission *line* in the spectrum of a moving astronomical source, and compare its wavelength with some zero-velocity reference. The first references were simply the wavelength scales in visual spectrographs. Angelo Secci, in Paris, and William Huggins, in London, both attempted visual measurements for the brighter stars during the period 1868–1876, with disappointing results. Probable errors for visual measurements were on the order of 30 km s^{-1}, a value similar to the actual velocities of most of the bright stars. James Keeler, at Lick Observatory in California, eventually was able to make precision visual measurements (errors of about 2–4 km s^{-1}), at about the same time (1888–1891) that astronomers at Potsdam first began photographing spectra. ***Spectrographs*** (with photographic recording) immediately proved vastly superior to ***spectroscopes***. Observers soon began recording ***comparison spectra***, usually from electrically activated iron arcs or hydrogen gas discharges, to provide a recorded wavelength scale. Figure 3.17 shows a photographic spectrum and comparison. A measuring engine (see Figure 3.14), a microscope whose stages are moved by screws equipped with micrometer read-outs, soon became essential for determining positions of the lines in the source spectrum relative to the lines in the comparison. In current practice, astronomers record spectra and comparisons digitally and compute shifts and velocities directly from the data.

Precise radial velocities. What limits the precision of a radial velocity measurement? We consider spectrometry in detail in Chapter 11. For now, just

Fig. 3.17 A conventional photographic spectrum. A stellar spectrum, with absorption lines, lies between two emission-line comparisons.

note that, since the important measurement is the physical location of spectral lines on the detector, an astronomer certainly would want to use a detector/spectrometer capable of showing as much detail as possible. The ***resolving power*** of a spectrograph is the ratio

$$R = \frac{\lambda}{\delta\lambda}$$

where $\delta\lambda$ is wavelength resolution (i.e. two narrow spectral lines that are closer than $\delta\lambda$ in wavelength will appear as a single line in the spectrogram). Limits to resolving power will be set by the design of the spectrograph, but also by the brightness of the object being investigated, and the size and efficiency of the telescope feeding the spectrograph. As is usual in astronomy, the most precise measurements can be made on the brightest objects.

Early spectroscopists soon discovered other limits to precision. They found that errors arose if a spectrograph had poor mechanical or thermal stability, or if the path taken by light from the source was not equivalent to the path taken by light from the comparison. New spectrograph designs improved resolving power, efficiency, stability, and the reliability of wavelength calibration. At the present time, random errors of less than 100 m s^{-1} in absolute stellar radial velocities are possible with the best optical spectrographs. At radio wavelengths, even greater precision is routine.

Greater precision is possible in differential measurements. Here the astronomer is concerned only with *changes* in the velocity of the object, not the actual value. Very precise optical work, for example, has been done in connection with searches for planets orbiting solar-type stars. The presence of a planet will cause the radial velocity of its star to vary as they both orbit the barycenter of the system. Precisions at a number of observatories now approach 3 m s^{-1} or better for differential measurements of brighter stars.

Large redshifts. When the radial velocity of the source is a considerable fraction of the speed of light, special relativity replaces Equation (3.4) with the correct version:

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0} = \frac{\left(1 - \beta^2\right)^{1/2}}{(1 - \beta)} - 1 \qquad (3.5)$$

where

$$\beta = \frac{v_R}{c} = \frac{1(z+1)^2 - 1}{1(z+1)^2 + 1}$$

Here z is called the ***redshift parameter***, or just the redshift. If the source moves away from the observer, both v_R and z are positive, and a spectral feature in the visual (yellow-green) will be shifted to longer wavelengths (i.e. towards the red). The spectrum is then said to be ***redshifted*** (even if the observed feature were a microwave line that was shifted to longer wavelengths and thus *away* from the red). Likewise, if the source moves towards the observer, v_R and z are negative, and the spectrum is said to be ***blueshifted.***

In an early result from the spectroscopy of non-stellar objects, Vesto Melvin Slipher, in 1914, noticed that the vast majority of the spiral nebulae (galaxies) had redshifted spectra. By 1931, Milton Humason and Edwin Hubble had recorded galaxy radial velocities up to 20,000 km s^{-1}, and were able to demonstrate that the redshift of a galaxy was directly proportional to its distance. Most astronomers interpret ***Hubble's law,***

$$v_R = H_0 d \tag{3.6}$$

as indicating that our Universe is expanding (the distances between galaxies are increasing). In Equation (3.6), it is customary to measure v in km s^{-1} and d in megaparsecs, so H_0, which is called the ***Hubble constant***, has units of km s^{-1} Mpc^{-1}. In these units, recent measurements of the Hubble constant fall in the range 67–77. Actually, the redshifts are not interpreted as due to the Doppler effect, but as the result of the expansion of space itself.

The object with the largest spectroscopic redshift (as of early 2007) is a galaxy, IOK-1, which has $z = 6.96$. You can expect additional detections in this range. Doppler's 1842 assumption that major components of the Universe have significant shifts in their spectra was quite correct after all.

Summary

- Coordinate systems can be characterized by a particular origin, reference plane, reference direction, and sign convention.
- Astronomical coordinates are treated as coordinates on the surface of a sphere. The laws of ***spherical trigonometry*** apply. Concepts:
  great circle *law of cosines* *law of sines*
- The geocentric terrestrial ***latitude and longitude*** system uses the equatorial plane and ***prime meridian*** as references. Concepts:
  geocentric latitude *geodetic latitude* *geographic latitude*
  Greenwich *polar motion*
- The altitude–azimuth system has its origin at the observer and uses the horizontal plane and geographic north as references. Concepts:
  vertical circle *zenith* *nadir*
  zenith distance *meridian* *diurnal motion*
  sidereal day
- The equatorial system of right ascension and declination locates objects on the celestial sphere. The Earth's equatorial plane and the vernal equinox are the references. This system rotates with respect to the altitude–azimuth system. Concepts:

(*continued*)

Summary (*cont.*)

celestial pole	*ecliptic*	*obliquity*
altitude of pole= observer's latitude	*upper meridian*	*circumpolar star*
	sidereal time	*hour circle*
transit		
hour angle		

- Astrometry establishes the positions of celestial objects. Positions are best transformed into the International Celestial Reference Frame (***ICRS***) which is independent of motions of the Earth. Concepts:

transit telescope	*meridian circle*	*VLBI*
HIPPARCOS	*Hipparchus*	*atmospheric refraction*
fundamental catalog	*Gaia*	*epoch*
precession	*nutation*	*apparent coordinates*
aberration of starlight	*ecliptic coordinates*	*J2000*
Galactic coordinates		

- Heliocentric stellar parallax is an effect that permits measurement of distances to nearby stars. Concepts:

astronomical unit (au)	*astronomical triangle*
parallax angle	*parsec (pc)*

$$p[\text{arcsec}] = \frac{a[\text{au}]}{r[\text{pc}]}$$

- Physicists define time in terms of the behavior of light, but practical time measurements have been historically tied to the rotation of the Earth. Concepts:

atomic clock	*local apparent solar time*
TAI second	*local mean solar time*
universal time	*coordinated universal time*
zone time	*Julian date*

- The tangential component of an object's velocity in the ICRS system gives rise to a change in angular position whose rate of change is called the proper motion.

$$v_T\left[\text{km s}^{-1}\right] = 4.74\frac{\mu[\text{arcsec yr}^{-1}]}{p[\text{arcsec}]}$$

- The radial component of an object's velocity can be measured by a shift in its spectrum due to the Doppler effect. Similar shifts are caused by the expansion of the universe. Concepts:

redshift parameter: $z = \Delta\lambda/\lambda \approx v_R/c$

spectroscopic resolving power (R) *relativistic Doppler effect*

Hubble's law: $v_R = H_0 d$

Exercises

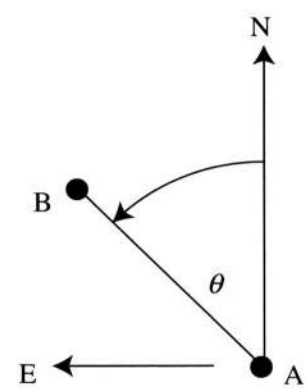
Each problem that I solved became a rule which served afterwards to solve other problems.

– Rene Descartes, *Discours de la Méthode* ..., 1637

1. Two objects differ in RA by an amount $\Delta\alpha$, and have declinations δ_1 and δ_2. Show that their angular separation, θ, is given by

$$\cos\theta = \sin\delta_1 \sin\delta_2 + \cos\delta_1 \cos\delta_2 \cos\Delta\alpha$$

2. Which city is closer to New York (74° W, 41° N): Los Angeles (118° W, 34° N) or Mexico City (99° W, 19° N)? By how much? (The radius of the Earth is 6300 km).
3. A. Kustner conducted one of the first systematic radial velocity studies. In 1905, he found that the velocity of stars in the ecliptic plane varied with an amplitude of $29.617 \pm .057$ km s^{-1} in the course of a sidereal year. Assume that the Earth's orbit is circular and use this information to derive the length (and uncertainty) of the au in kilometers.
4. Position angles are measured from north through east on the sky. For example, the figure at right shows a double star system in which component B is located in position angle θ with respect to component A. The two have an angular separation of r seconds of arc. If component A has equatorial coordinates (α, δ), and B has coordinates ($\alpha + \Delta\alpha$, $\delta + \Delta\delta$), derive expressions for $\Delta\alpha$ and $\Delta\delta$.



5. The field of view of the Vassar 32-inch CCD camera is a square 1000 seconds of arc on each side. Fill in the width of the field in the RA coordinate (i.e. in H:M:S units) when the telescope is pointed at declinations listed in the table:

Declination (degrees)	Width of field (minutes:seconds of RA)
0	1:06.7
20	
40	
60	
70	
80	
85	

6. The winter solstice (December 22) is the date of the longest night of the year in the northern hemisphere. However, the date of the earliest sunset in the northern hemisphere occurs much earlier in the month (at about 16:35 zone time on December 8 for longitude 0 degrees and latitude 40 degrees N). Examine the curve for the equation of time and suggest why this might be the case. Explain how this observation would depend upon one's exact longitude within a time zone.
7. On the date of the winter solstice, what are the approximate local sidereal times and local apparent solar times at sunset? (Assume 40° N latitude and use a celestial sphere.)

8. A certain supernova remnant in our galaxy is an expanding spherical shell of glowing gas. The angular diameter of the remnant, as seen from Earth, is 22.0 arcsec. The parallax of the remnant is known to be 4.17 mas from space telescope measurements. Compute its distance in parsecs and radius in astronomical units.
9. An astronomer obtains a spectrum of the central part of the above remnant, which shows emission lines. Close examination of the line due to hydrogen near wavelength 656 nm reveals that it is actually double. The components, presumably from the front and back of the shell, are separated by 0.160 nanometers. (a) With what velocity is the nebula expanding? (b) Assuming this has remained constant, estimate the age of the remnant. (c) The astronomer compares images of the remnant taken 60 years apart, and finds that the nebula has grown in diameter from 18.4 to 22.0 arcsec. Use this data to make a new computation for the distance of the remnant independent of the parallax.
10. In 1840, the estimated value of the au, 1.535×10^8 km, was based upon Encke's 1824 analysis of the observations of the transits of Venus in 1761 and 1769. Encke's result should have been accorded a relative uncertainty of around 5%. If Bessel's (1838) parallax for 61 Cygni was 0.32 ± 0.04 arcsec, compute the distance and the total relative uncertainty in the distance to this star, in kilometers, from the data available in 1840. If the presently accepted value for the parallax is 287.1 ± 0.5 mas, compute the modern estimate of the distance, again in kilometers, and its uncertainty.
11. The angular diameter of the Sun is 32 arc minutes when it is at the zenith. Using the table below (you will need to interpolate), plot a curve showing the apparent shape of the Sun as it sets. You should plot the ellipticity of the apparent solar disk as a function of the elevation angle of the lower limb, for elevations between 0 and 10 degrees. (If a and b are the semi-major and semi-minor axes of an ellipse, its ellipticity, ε, is $(a-b)/a$. The ellipticity varies between 0 and 1.) Is your result consistent with your visual impression of the setting Sun?

Apparent zenith distance (degrees)	75	80	83	85	86	87	88	89	89.5	90
Atmospheric refraction (arcsec)	215	320	445	590	700	860	1103	1480	1760	2123

12. The Foggy Bottom Observatory has discovered an unusual object near the ecliptic, an object some students suggest is a very nearby sub-luminous star, and others think is a trans-Neptunian asteroid. The object was near opposition on the date of discovery.

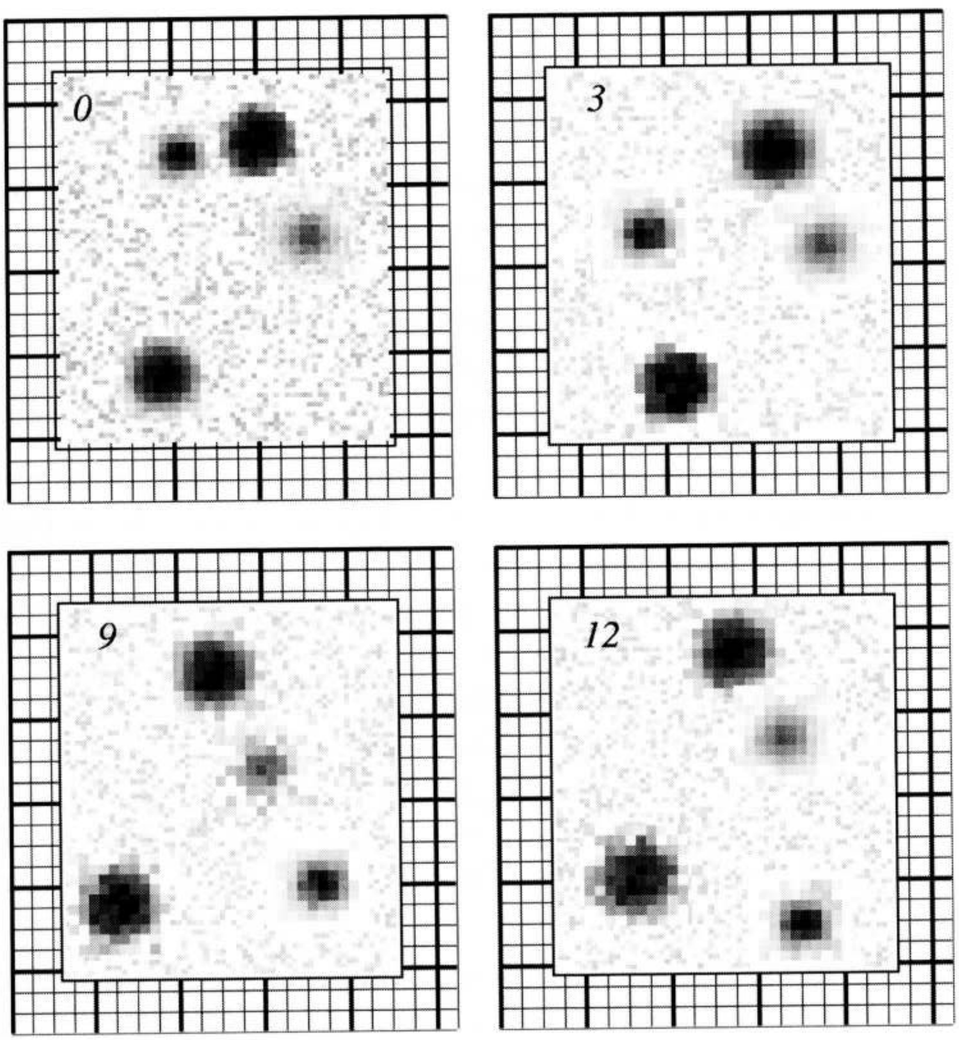
    Below are sketches of four CCD images of this object, taken 0, 3, 9 and 12 months after discovery. Sketches are oriented so that ecliptic longitude is in the horizontal direction. The small squares in the grid surrounding each frame measure 250 mas × 250 mas. Note that the alignment of the grid and stars varies from frame to frame.

(a) Why is there no frame 6 months after discovery?

(b) Compute the proper motion, parallax, and distance to this object.

(c) Is it a star or an asteroid? Explain your reasoning.

(d) Compute its tangential velocity.



13. Current telescopes can detect stellar objects with apparent magnitudes as faint as $V = 24$ with good precision. What is the greatest distance that a supernova of magnitude –20 can be detected? Compute the expected redshift parameter of such an object.
14. The distances (there are actually several definitions of "distance" in an expanding universe) corresponding to redshift parameters larger than 1 actually depend on several cosmological parameters, not the simple relations in Equations (3.5) and (3.6). For example, the time light has taken to travel from the galaxy IOK-1 ($z = 6.96$) to us, using current values of these parameters, is about 12.88 Gyr. Compare this with the light travel time computed naively from Equations (3.5) and (3.6).