

where σ_d is the average distance that any single electron moves. Therefore,

$$\sigma_V = \frac{e}{L} \sqrt{N} \frac{\sigma_d}{t_0} R. \quad (3.17)$$

Now N is the total number of conduction electrons in the resistor times the number of walks in time t_0 , so

$$N = (nAL) \times \frac{t_0}{\tau} = \frac{nALt_0}{\tau},$$

where n is the number density of conduction electrons and τ is the time between collisions of a *single* electron. The fluctuation in the motion of a single electron is

$$\sigma_d^2 = \langle d^2 \rangle = \langle v_x^2 \tau^2 \rangle = \langle v_x^2 \rangle \tau^2,$$

and this is what we connect to temperature by $\langle E \rangle = \frac{1}{2} m \langle v_x^2 \rangle = \frac{1}{2} kT$, where m is the mass of an electron and we note that motion is only in one dimension. The factor k is Boltzmann's constant which defines the fundamental relationship between temperature and internal energy. Therefore

$$\sigma_d^2 = \frac{kT \tau^2}{m}.$$

We note that (see Eq. (2.14))

$$\frac{L}{A} \frac{2m}{ne^2 \tau} = \frac{L}{A} \rho = R,$$

where ρ is the resistivity.⁵

Finally, put this all into Eq. (3.17) to get

$$\begin{aligned} \sigma_V^2 &= \frac{e^2}{L^2} N \frac{\sigma_d^2}{t_0^2} R^2 \\ &= \frac{e^2}{L^2} \frac{nALt_0}{\tau} \frac{kT \tau^2}{mt_0^2} R^2 \\ &= \frac{A}{L} \frac{ne^2 \tau}{m} \frac{kT}{t_0} R^2, \end{aligned}$$

⁵The definition of τ used here differs from that used in Section 2.2 by a factor of 2. That is because we are dealing with a single electron.

or

$$\langle V^2 \rangle = \frac{2kTR}{t_0}. \quad (3.18)$$

It is customary, however, to express the noise using the equivalent bandwidth $\Delta\nu = 1/2t_0$. Therefore, we have

$$\langle V^2 \rangle = 4kTR \Delta\nu. \quad (3.19)$$

In order to measure the voltage V , we will need to amplify or at least process the signal in some way. Let $g(\nu)$ be the gain of this processing circuit at frequency ν . Then the output voltage fluctuation $d\langle V^2 \rangle$ integrated over some small frequency range $d\nu$ is given by

$$d\langle V^2 \rangle = 4kTRg^2(\nu) d\nu.$$

Measurements are made by integrating the signal over a relatively large bandwidth $\Delta\nu$. This bandwidth is typically determined by the gain function $g(\nu)$, which is large only over some finite frequency range. We therefore obtain the expression

$$\langle V^2 \rangle = 4kTRG^2 \Delta\nu, \quad (3.20)$$

where G and $\Delta\nu$ are constants defined by

$$G^2 \Delta\nu \equiv \int_0^\infty g^2(\nu) d\nu. \quad (3.21)$$

3.6.2. Measurements

We will measure the Johnson noise in a series of resistors, and use the result to determine a value for Boltzmann's constant k .

The setup is shown schematically in Fig. 3.22. The voltage across the resistor R is immediately processed by an "amplifier," which essentially multiplies this voltage by a function $g(\nu)$. The output of the amplifier is measured using a digital oscilloscope. You will use the oscilloscope to measure $\langle V^2 \rangle$, given by Eq. (3.20). By changing the value of R (simply by changing resistors), you measure $\langle V^2 \rangle$ as a function of R , and the result should be a straight line. The slope of the line is just $4kTG^2 \Delta\nu$, so once you have calibrated the gain function of the amplifier, you can get k . (You can assume the resistor is at room temperature.)

