

where  $\sigma_d$  is the average distance that any single electron moves. Therefore,

$$\sigma_V = \frac{e}{L} \sqrt{N} \frac{\sigma_d}{t_0} R. \quad (3.17)$$

Now  $N$  is the total number of conduction electrons in the resistor times the number of walks in time  $t_0$ , so

$$N = (nAL) \times \frac{t_0}{\tau} = \frac{nALt_0}{\tau},$$

where  $n$  is the number density of conduction electrons and  $\tau$  is the time between collisions of a *single* electron. The fluctuation in the motion of a single electron is

$$\sigma_d^2 = \langle d^2 \rangle = \langle v_x^2 \tau^2 \rangle = \langle v_x^2 \rangle \tau^2,$$

and this is what we connect to temperature by  $\langle E \rangle = \frac{1}{2} m \langle v_x^2 \rangle = \frac{1}{2} kT$ , where  $m$  is the mass of an electron and we note that motion is only in one dimension. The factor  $k$  is Boltzmann's constant which defines the fundamental relationship between temperature and internal energy. Therefore

$$\sigma_d^2 = \frac{kT \tau^2}{m}.$$

We note that (see Eq. (2.14))

$$\frac{L}{A} \frac{2m}{ne^2 \tau} = \frac{L}{A} \rho = R,$$

where  $\rho$  is the resistivity.<sup>5</sup>

Finally, put this all into Eq. (3.17) to get

$$\begin{aligned} \sigma_V^2 &= \frac{e^2}{L^2} N \frac{\sigma_d^2}{t_0^2} R^2 \\ &= \frac{e^2}{L^2} \frac{nALt_0}{\tau} \frac{kT \tau^2}{mt_0^2} R^2 \\ &= \frac{A}{L} \frac{ne^2 \tau}{m} \frac{kT}{t_0} R^2, \end{aligned}$$

<sup>5</sup>The definition of  $\tau$  used here differs from that used in Section 2.2 by a factor of 2. That is because we are dealing with a single electron.

or

$$\langle V^2 \rangle = \frac{2kTR}{t_0}. \quad (3.18)$$

It is customary, however, to express the noise using the equivalent bandwidth  $\Delta\nu = 1/2t_0$ . Therefore, we have

$$\langle V^2 \rangle = 4kTR \Delta\nu. \quad (3.19)$$

In order to measure the voltage  $V$ , we will need to amplify or at least process the signal in some way. Let  $g(\nu)$  be the gain of this processing circuit at frequency  $\nu$ . Then the output voltage fluctuation  $d\langle V^2 \rangle$  integrated over some small frequency range  $d\nu$  is given by

$$d\langle V^2 \rangle = 4kTR g^2(\nu) d\nu.$$

Measurements are made by integrating the signal over a relatively large bandwidth  $\Delta\nu$ . This bandwidth is typically determined by the gain function  $g(\nu)$ , which is large only over some finite frequency range. We therefore obtain the expression

$$\langle V^2 \rangle = 4kTR G^2 \Delta\nu, \quad (3.20)$$

where  $G$  and  $\Delta\nu$  are constants defined by

$$G^2 \Delta\nu \equiv \int_0^\infty g^2(\nu) d\nu. \quad (3.21)$$

### 3.6.2. Measurements

We will measure the Johnson noise in a series of resistors, and use the result to determine a value for Boltzmann's constant  $k$ .

The setup is shown schematically in Fig. 3.22. The voltage across the resistor  $R$  is immediately processed by an "amplifier," which essentially multiplies this voltage by a function  $g(\nu)$ . The output of the amplifier is measured using a digital oscilloscope. You will use the oscilloscope to measure  $\langle V^2 \rangle$ , given by Eq. (3.20). By changing the value of  $R$  (simply by changing resistors), you measure  $\langle V^2 \rangle$  as a function of  $R$ , and the result should be a straight line. The slope of the line is just  $4kTG^2 \Delta\nu$ , so once you have calibrated the gain function of the amplifier, you can get  $k$ . (You can assume the resistor is at room temperature.)

